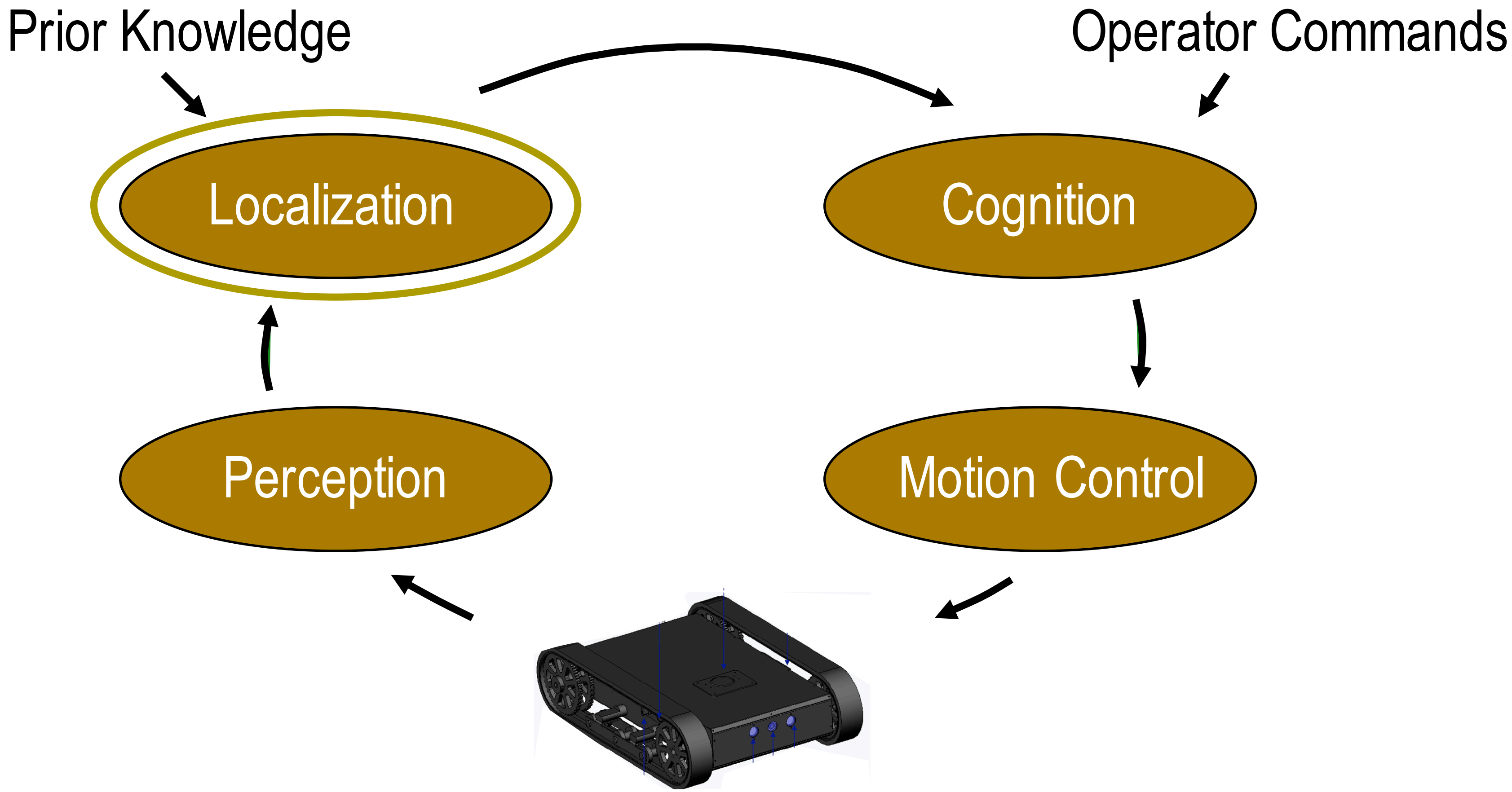
E160 – Lecture 12
Autonomous Robot Navigation

Instructor: Chris Clark

Semester: Spring 2018

Figures courtesy of Probabilistic Robotics (Thrun et. Al.)

Control Structures
Planning Based Control



SLAM

- Introduction to SLAM
- Landmark based SLAM
- Occupancy Grid based SLAM

Methods

- Mapping Problem
  - Determine the state of the environment given a known robot state.
- Localization Problem
  - Determine the state of a robot given a known environment state.
- SLAM – Simultaneous Localization and Mapping
  - Simultaneously determine the state of a robot and state of the environment.

SLAM

- Full SLAM
  - Estimates entire path of robot and across all time.

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$

- On Line SLAM
  - Estimates current pose of the robot and map.
  - Integrations typically done one at a time

$$p(x_t, m \mid z_{1:t}, u_{1:t})$$

SLAM

- Introduction to SLAM
- Landmark based SLAM
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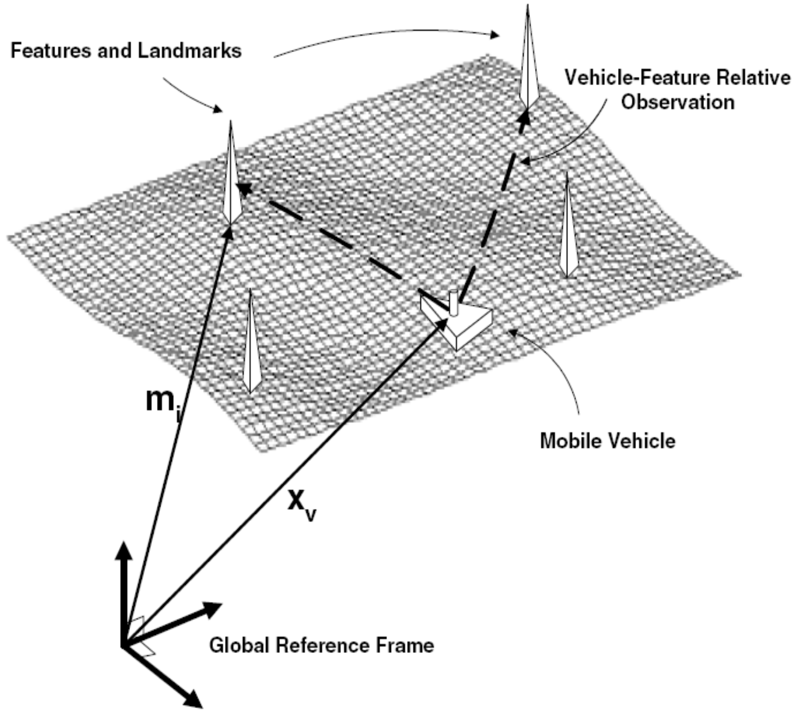
SLAM

- Landmark based SLAM
  - Features
    - **Observable** parts or characteristics of objects in the environment.
    - E.g. corners, colors, walls, etc.
  - Landmarks
    - **Static** and **easily recognizable** features**.**
    - E.g. Orange cones

SLAM

- Landmark based SLAM
  - Given:
    - The robot's odometry **u**
    - Observations of nearby features **z**
  - Estimate:
    - Robot States **x**
    - Landmark States **M**



EKF SLAM

- To start, lets recall our EKF Localization…

EKF Localization

- In our example, the state vector to be estimated, $\mathbf{x}$, was a 3x1 vector

  e.g.

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

  - Associated Covariance, $\mathbf{P}$

$$\mathbf{P} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} \end{bmatrix}$$

EKF Localization

Prediction

1. $\mathbf{x}'_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t)$
2. $\mathbf{P}'_t = \mathbf{F}_{x,t-1} \mathbf{P}_{t-1} \mathbf{F}_{x,t-1}^T + \mathbf{F}_{u,t} \mathbf{Q}_t \mathbf{F}_{u,t}^T$

Correction

3. $\mathbf{z}^i_{exp,t} = h^i(\mathbf{x}'_t, \mathbf{M})$
4. $\mathbf{v}_t = \mathbf{z}_t - \mathbf{z}_{exp,t}$
5. $\Sigma_{IN,t} = \mathbf{H}^i_{x',t} \mathbf{P}'_t \mathbf{H}^i_{x',t}{}^T + \mathbf{R}^i_t$
6. $\mathbf{K}_t = \mathbf{P}'_t \mathbf{H}_{x',t}^T (\Sigma_{IN,t})^{-1}$
7. $\mathbf{x}_t = \mathbf{x}'_t + \mathbf{K}_t \mathbf{v}_t$
8. $\mathbf{P}_t = \mathbf{P}'_t - \mathbf{K}_t \Sigma_{IN,t} \mathbf{K}_t^T$

EKF SLAM

- In SLAM, the state vector to be estimated

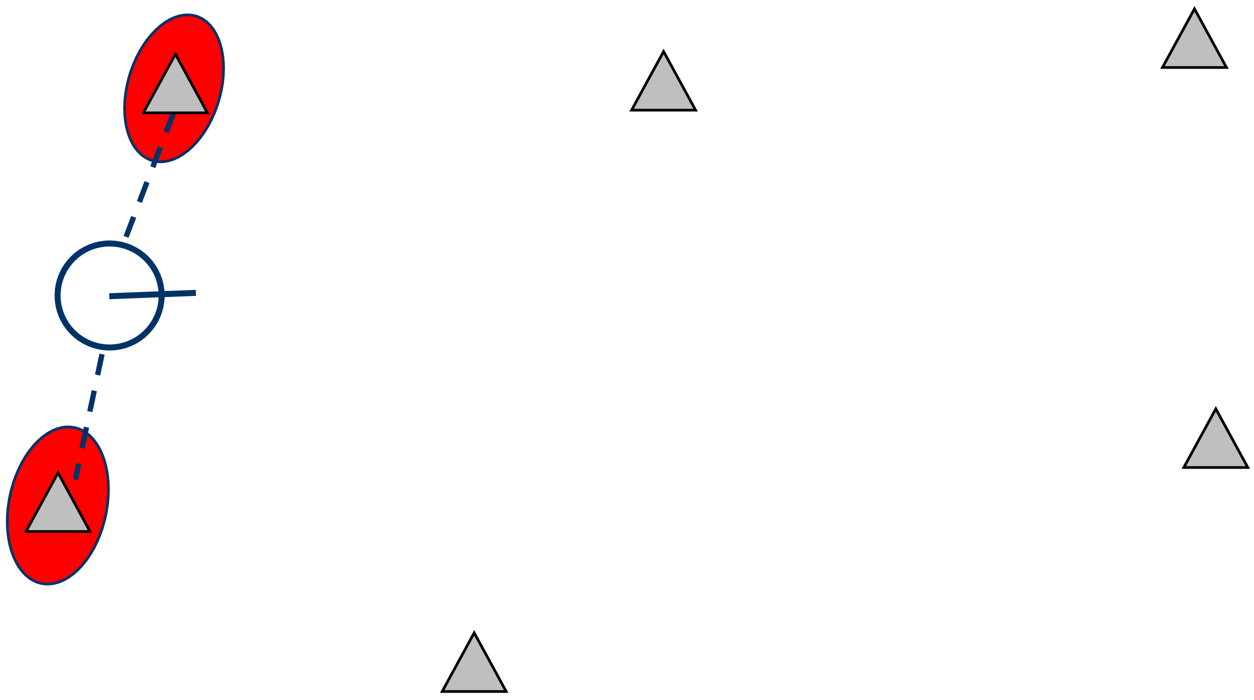
$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \theta \\ x_{f1} \\ y_{f1} \\ \dots \\ x_{fN} \\ y_{fN} \end{bmatrix}$$

EKF SLAM

- The covariance Matrix $\mathbf{P}$

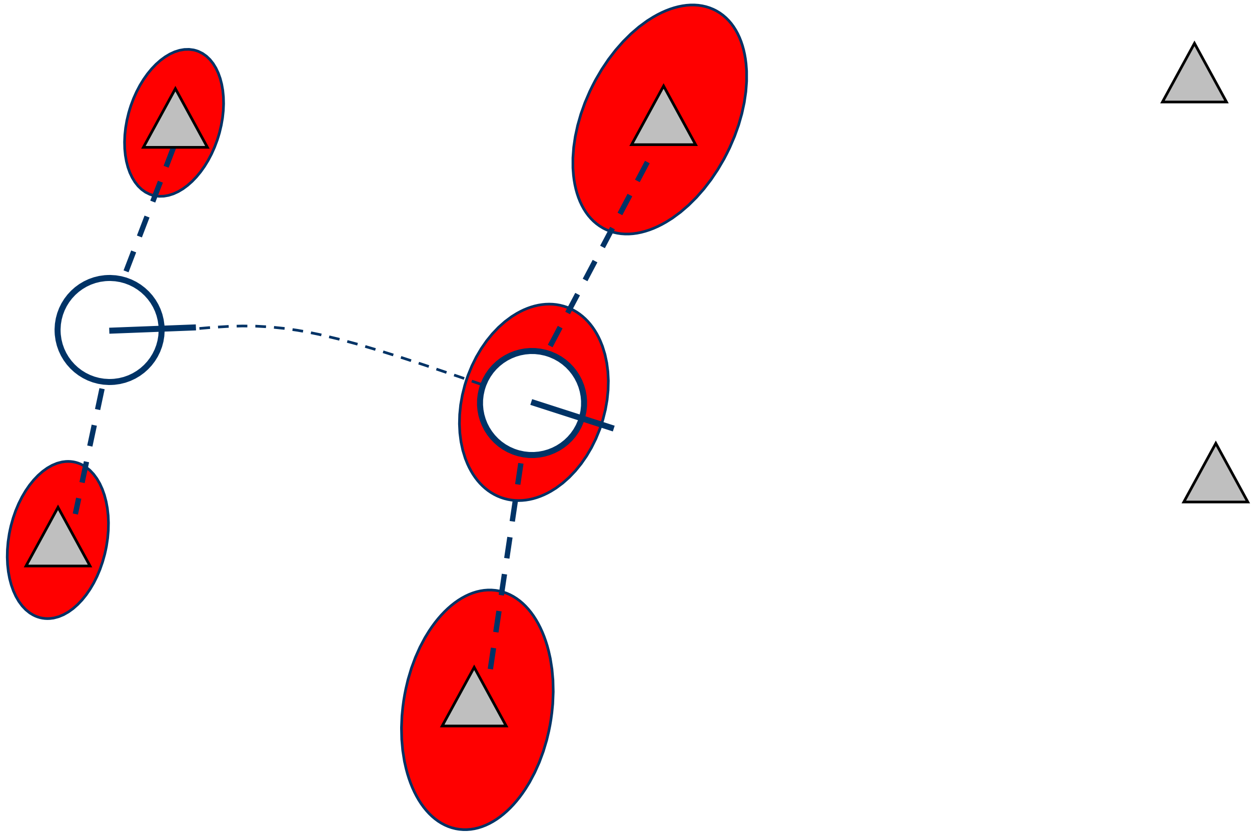
$$\mathbf{P} =$$

Landmark Based Example



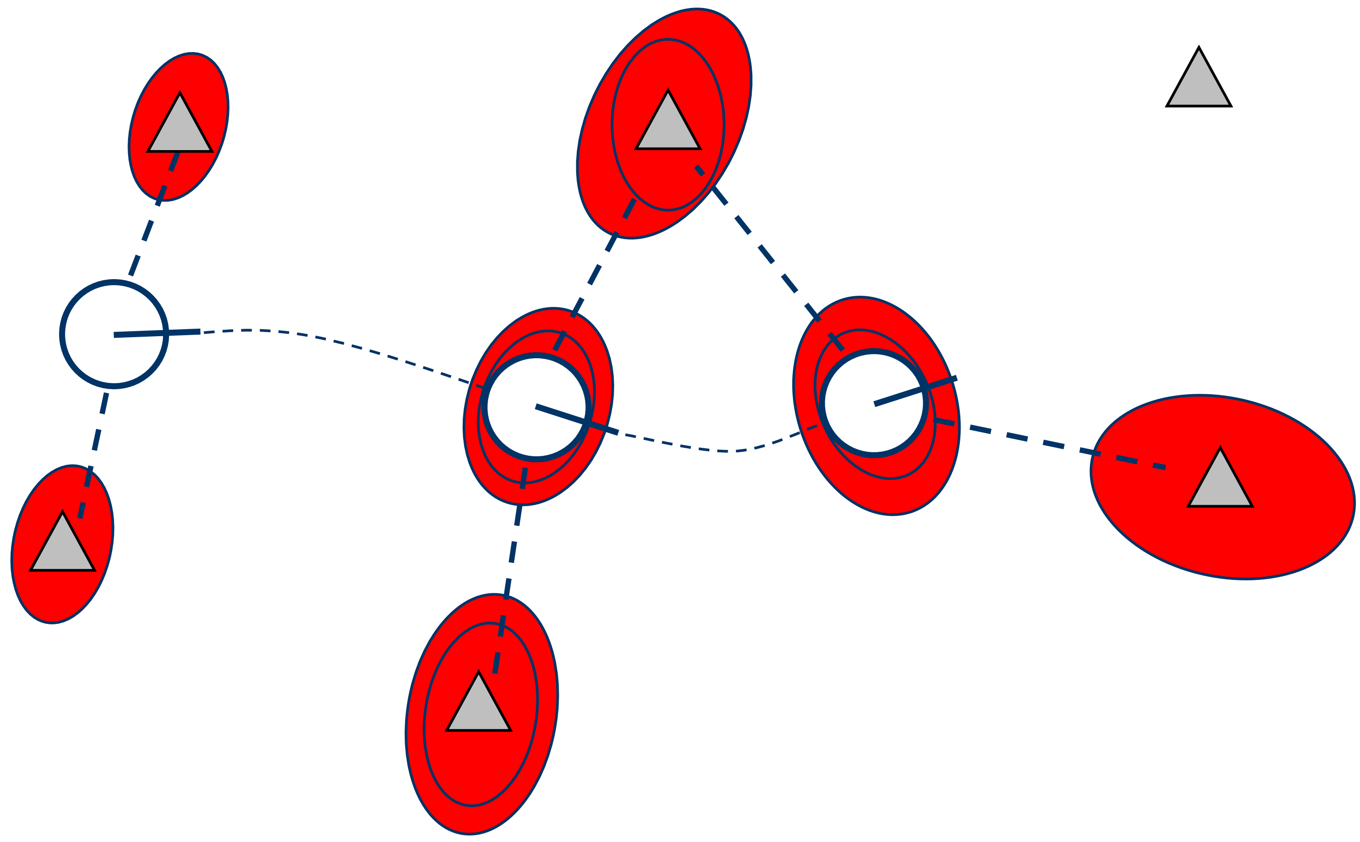
Robot path error correlates errors in the map

Landmark Based Example



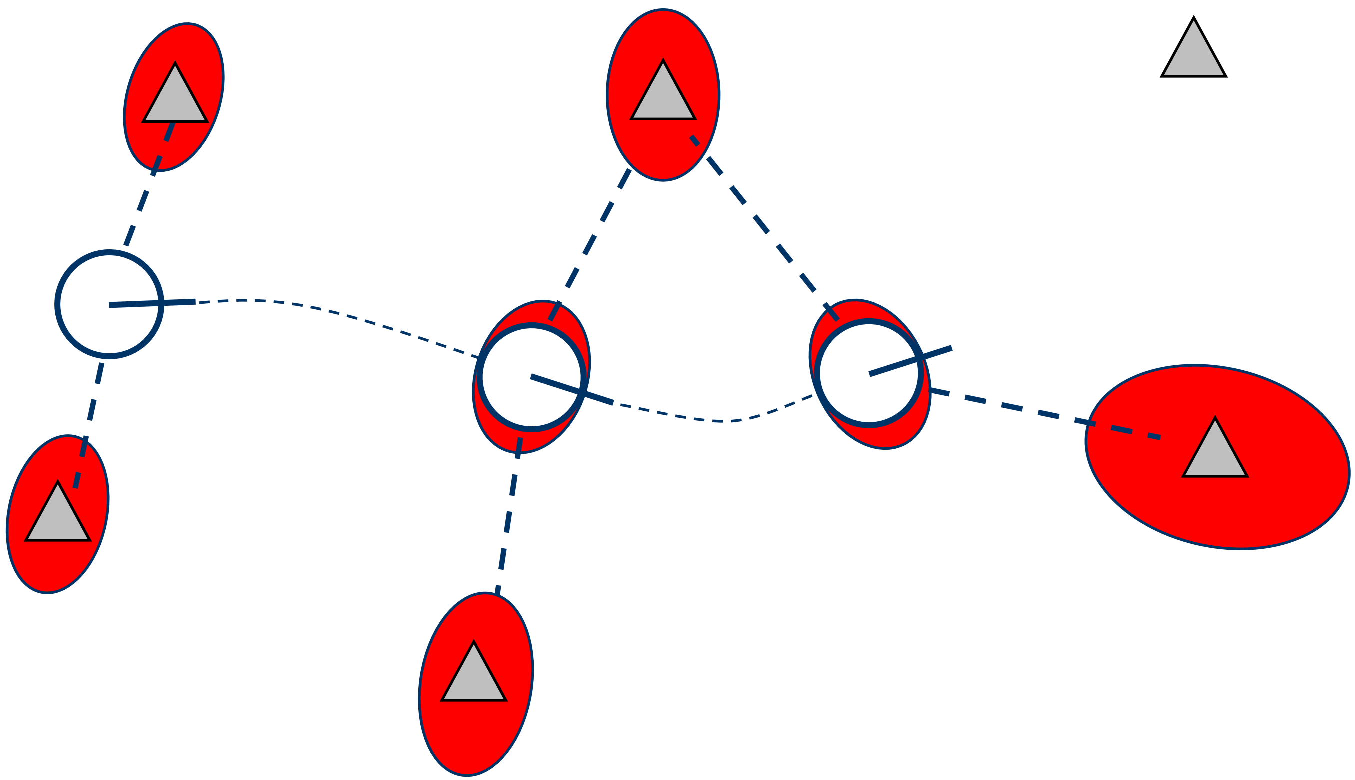
Robot path error correlates errors in the map

Landmark Based Example



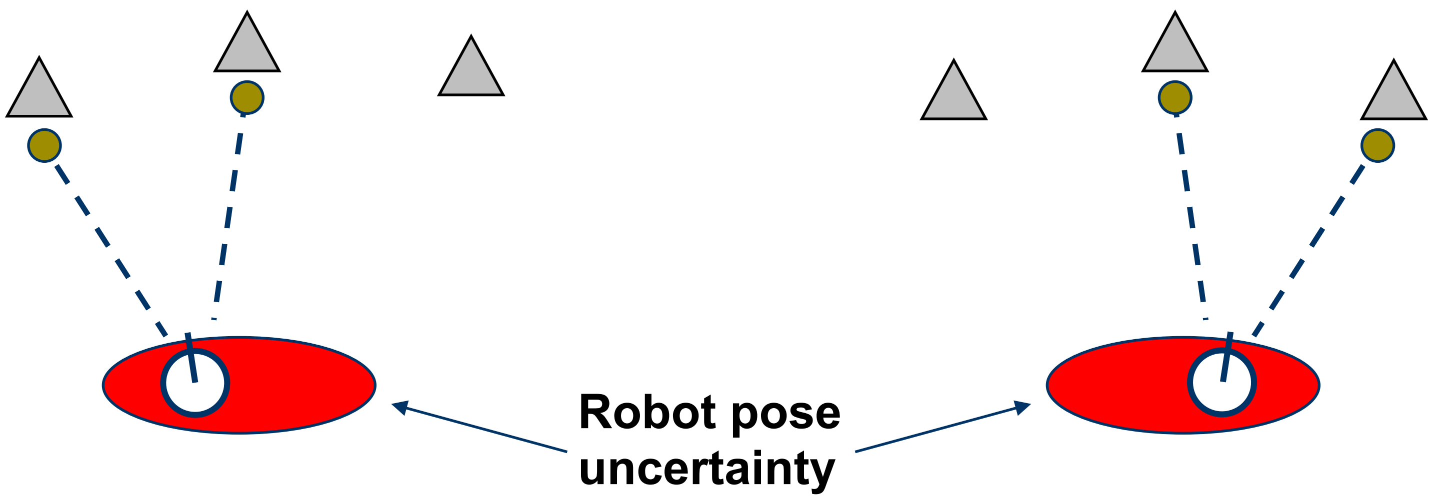
Robot path error correlates errors in the map

Landmark Based Example



Robot path error correlates errors in the map

Why is SLAM a hard problem?



- The matching between observations and landmarks is unknown
- Wrong data associations can have catastrophic consequences

EKF SLAM

Prediction

1. $\mathbf{x}'_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t)$
2. $\mathbf{P}'_t = \mathbf{F}_{x,t-1} \mathbf{P}_{t-1} \mathbf{F}_{x,t-1}^T + \mathbf{F}_{u,t} \mathbf{Q}_t \mathbf{F}_{u,t}^T$

Correction

3. $\mathbf{z}^i_{exp,t} = h^i(\mathbf{x}'_t)$
4. $\mathbf{v}_t = \mathbf{z}_t - \mathbf{z}_{exp,t}$
5. $\Sigma_{IN,t} = \mathbf{H}^i_{x',t} \mathbf{P}'_t \mathbf{H}^i_{x',t}{}^T + \mathbf{R}^i_t$
6. $\mathbf{K}_t = \mathbf{P}'_t \mathbf{H}_{x',t}^T (\Sigma_{IN,t})^{-1}$
7. $\mathbf{x}_t = \mathbf{x}'_t + \mathbf{K}_t \mathbf{v}_t$
8. $\mathbf{P}_t = \mathbf{P}'_t - \mathbf{K}_t \Sigma_{IN,t} \mathbf{K}_t^T$

Prediction Step

- Localization Motion model

$$\mathbf{x}'_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t) = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \Delta s_t \cos(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta s_t \sin(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta\theta_t \end{bmatrix}$$

Prediction Step

- SLAM Motion Model

$$\mathbf{x}'_t = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \\ x_{f1\,t-1} \\ y_{f1\,t-1} \\ \dots \\ x_{fN\,t-1} \\ y_{fN\,t-1} \end{bmatrix} + \begin{bmatrix} \Delta s_t \cos(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta s_t \sin(\theta_{t-1} + \Delta\theta_t/2) \\ \Delta\theta_t \\ 0 \\ 0 \\ \dots \\ 0 \\ 0 \end{bmatrix}$$

Prediction Step

- Covariance
  - Recall, we linearize the motion model f to obtain

$$\mathbf{P}'_t = \mathbf{F}_{x,t-1}\,\mathbf{P}_{t-1}\mathbf{F}_{x,t-1}^T + \mathbf{F}_{u,t}\,\mathbf{Q}_t\,\mathbf{F}_{u,t}^T$$

where

$\mathbf{Q}_t$ = *Motion Error Covariance Matrix*

$\mathbf{F}_{x,t-1}$ = *Derivative of f with respect to state* $\mathbf{x}_{t-1}$

$\mathbf{F}_{u,t}$ = *Derivative of f with respect to control* $\mathbf{u}_t$

Prediction Step

- Covariance

$$\mathbf{P}'_t = \mathbf{F}_{x,t-1}\,\mathbf{P}_{t-1}\mathbf{F}_{x,t-1}^{T} + \mathbf{F}_{u,t}\,\mathbf{Q}_t\,\mathbf{F}_{u,t}^{T}$$

Prediction Step

- Covariance

$$F_{x,t-1} = \begin{bmatrix} dx_t/dx_{t-1} & dx_t/dy_{t-1} & dx_t/d\theta_{t-1} & dx_t/dx_{f1t-1} & \dots & dx_t/dy_{fNt-1} \\ dy_t/dx_{t-1} & dy_t/dy_{t-1} & dy_t/d\theta_{t-1} & dy_t/dx_{f1t-1} & \dots & dy_t/dy_{fNt-1} \\ d\theta_t/dx_{t-1} & d\theta_t/dy_{t-1} & d\theta_t/d\theta_{t-1} & d\theta_t/dx_{f1t-1} & \dots & d\theta_t/dy_{fNt-1} \\ dx_{f1t}/dx_{t-1} & dx_{f1t}/dy_{t-1} & dx_{f1t}/d\theta_{t-1} & dx_{f1t}/dx_{f1t-1} & \dots & dx_{f1t}/dy_{fNt-1} \\ dy_{f1t}/dx_{t-1} & dy_{f1t}/dy_{t-1} & dy_{f1t}/d\theta_{t-1} & dy_{f1t}/dx_{f1t-1} & \dots & dy_{f1t}/dy_{fNt-1} \\ & \dots & & \dots & & \\ dy_{fNt}/dx_{t-1} & dy_{fNt}/dy_{t-1} & dy_{fNt}/d\theta_{t-1} & dy_{fNt}/dx_{f1t-1} & \dots & dy_{fNt}/dy_{fNt-1} \end{bmatrix}$$

Prediction Step

- Covariance

$$P'_t = F_{x,t-1} P_{t-1} F_{x,t-1}^T + \mathbf{F}_{u,t} \mathbf{Q}_t \mathbf{F}_{u,t}^T$$

$$\mathbf{Q}_t = \begin{bmatrix} k\,|\Delta s_{r,t}| & 0 \\ 0 & k\,|\Delta s_{l,t}| \end{bmatrix}$$

$$\mathbf{F}_{u,t} = \begin{bmatrix} df/d\Delta s_{r,t} & df/d\Delta s_{l,t} \end{bmatrix}$$

Prediction Step

- Covariance

$$\mathbf{F}_{\boldsymbol{u},\boldsymbol{t}} = \begin{bmatrix} dx_t/d\Delta s_{r,t} & dx_t/d\Delta s_{l,t} \\ dy_t/d\Delta s_{r,t} & dy_t/d\Delta s_{l,t} \\ d\theta_t/d\Delta s_{r,t} & d\theta_t/d\Delta s_{l,t} \\ dx_{f1t}/d\Delta s_{r,t} & dx_{f1t}/d\Delta s_{l,t} \\ dy_{f1t}/d\Delta s_{r,t} & dy_{f1t}/d\Delta s_{l,t} \\ \multicolumn{2}{c}{\cdots} \\ dy_{fNt}/d\Delta s_{r,t} & dy_{fNt}/d\Delta s_{l,t} \end{bmatrix}$$

EKF SLAM

Prediction

1. $\mathbf{x}'_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t)$
2. $\mathbf{P}'_t = \mathbf{F}_{x,t-1} \mathbf{P}_{t-1} \mathbf{F}_{x,t-1}^T + \mathbf{F}_{u,t} \mathbf{Q}_t \mathbf{F}_{u,t}^T$

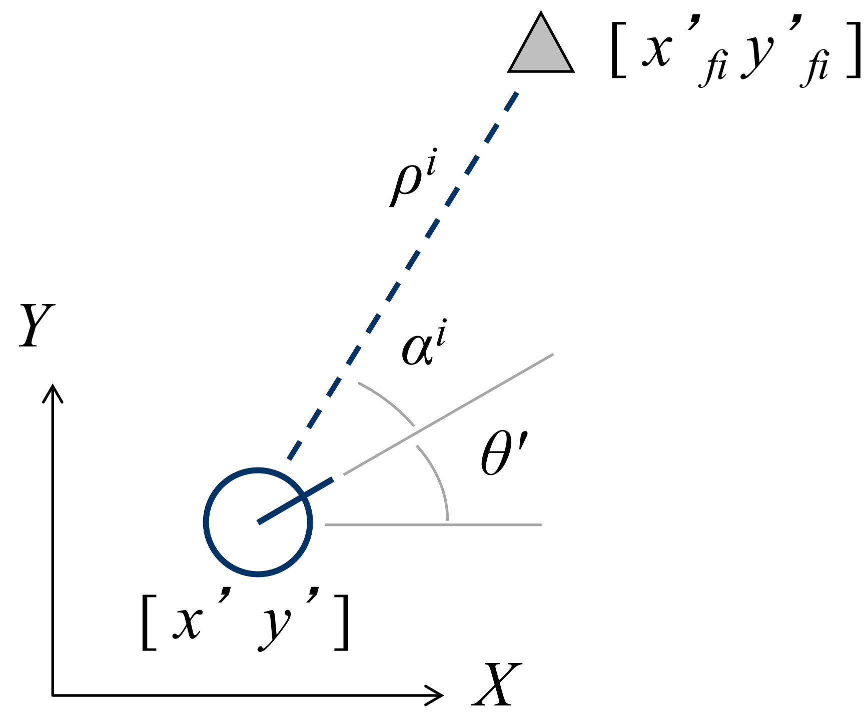
Correction

3. $\mathbf{z}^i_{exp,t} = h^i(\mathbf{x}'_t)$
4. $\mathbf{v}_t = \mathbf{z}_t - \mathbf{z}_{exp,t}$
5. $\Sigma^i_{IN,t} = \mathbf{H}^i_{x',t} \mathbf{P}'_t \mathbf{H}^i_{x',t}{}^T + \mathbf{R}^i_t$
6. $\mathbf{x}_t = \mathbf{x}'_t + \mathbf{K}_t \mathbf{v}_t$
7. $\mathbf{P}_t = \mathbf{P}'_t - \mathbf{K}_t \Sigma_{IN,t} \mathbf{K}_t^T$
8. $\mathbf{K}_t = \mathbf{P}'_t \mathbf{H}_{x',t}^T (\Sigma_{IN,t})^{-1}$

Correction Step

- Measurement of i^{th} landmark

$$\mathbf{z}^{i}_{t} = \begin{bmatrix} \alpha^{i}_{t} \\ \rho^{i}_{t} \end{bmatrix}$$



Correction Step

- Expected Measurement calculation

$$\mathbf{z}^{i}_{exp,t} = \begin{bmatrix} \alpha^{i}_{exp,t} \\ \rho^{i}_{exp,t} \end{bmatrix}$$

$$= h^{i}(\mathbf{x}'_{t})$$

$$= \begin{bmatrix} atan2(y_{fi} - y'_{t}, x_{fi} - x'_{t}) - \theta'_{t} \\ ((y_{fi} - y'_{t})^{2} + (x_{fi} - x'_{t})^{2})^{0.5} \end{bmatrix}$$

Correction Step

- Innovation calculation

$$\mathbf{v}^i_t = \mathbf{z}^i_t - \mathbf{z}^i_{exp,t}$$

$$= \begin{bmatrix} \alpha^i_t - \alpha^i_{exp,t} \\ \rho^i_t - \rho^i_{exp,t} \end{bmatrix}$$

Correction Step

- Innovation covariance calculation

$$\boldsymbol{\Sigma}^{i}_{IN,t} = \mathbf{H}^{i}_{x',t}\,\mathbf{P}'_{t}\,\mathbf{H}^{i\,T}_{x',t} + \mathbf{R}^{i}_{t}$$

where

$\mathbf{R}^{i}_{t}$ = *Feature Measurement Error Covariance Matrix*

$\mathbf{H}^{i}_{x',t}$ = *Derivative of h with respect to state* $\mathbf{x}'_{t}$

Correction Step

- Innovation covariance calculation

$$\Sigma^{i}_{IN,t} = \mathbf{H}^{i}_{x',t}\mathbf{P}'_{t}\mathbf{H}^{i\,T}_{x',t} + \mathbf{R}^{i}_{t}$$

$$\mathbf{H}^{i}_{x',t} = \begin{bmatrix} d\alpha^{i}_{exp,t}/dx'_{t} & d\alpha^{i}_{exp,t}/dy'_{t} & d\alpha^{i}_{exp,t}/d\theta'_{t} & d\alpha^{i}_{exp,t}/dx'_{f1t} & \ldots & d\alpha^{i}_{exp,t}/dy'_{fNt} \\ d\rho^{i}_{exp,t}/dx'_{t} & d\rho^{i}_{exp,t}/dy'_{t} & d\rho^{i}_{exp,t}/d\theta'_{t} & d\rho^{i}_{exp,t}/dx'_{f1t} & \ldots & d\rho^{i}_{exp,t}/dy'_{fNt} \end{bmatrix}$$

Correction Step

- Innovation covariance calculation

$$\Sigma^{i}_{IN,t} = \mathbf{H}^{i}_{x',t}\mathbf{P}'_{t}\mathbf{H}^{i\;T}_{x',t} + \mathbf{R}^{i}_{t}$$

$$\mathbf{R}^{i}_{t} = \begin{bmatrix} {\sigma^{i}_{\alpha}}^{2} & 0 \\ 0 & {\sigma^{i}_{\rho}}^{2} \end{bmatrix}$$

Correction Step

- For N landmarks …

$$\mathbf{z}_t = [\mathbf{z}^1_t \quad \mathbf{z}^2_t \ \ldots \quad \mathbf{z}^N_t]^T$$

$$\mathbf{z}_{exp,t} = [\mathbf{z}^1_{exp,t} \quad \mathbf{z}^2_{exp,t} \ \ldots \quad \mathbf{z}^N_{exp,t}]^T$$

Correction Step

- For N features…

$$\mathbf{v}_t = \mathbf{z}_t - \mathbf{z}_{exp,t}$$

$$= [\mathbf{v}^1_t \quad \mathbf{v}^2_t \ldots \quad \mathbf{v}^N_t]^T$$

Correction Step

- For N features …

$$\mathbf{H}_{x',t} = \begin{bmatrix} \mathbf{H}^{1}_{x',t} \\ \mathbf{H}^{2}_{x',t} \\ \\ \dots \\ \mathbf{H}^{N}_{x',t} \end{bmatrix}$$

Correction Step

- For N features …

$$\mathbf{\Sigma}_{IN,t} = \mathbf{H}_{x',t}\,\mathbf{P}_t\,\mathbf{H}_{x',t}^{T} + \mathbf{R}_t$$

EKF SLAM

Prediction

1. $\mathbf{x}'_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t)$
2. $\mathbf{P}'_t = \mathbf{F}_{x,t-1} \mathbf{P}_{t-1} \mathbf{F}_{x,t-1}^T + \mathbf{F}_{u,t} \mathbf{Q}_t \mathbf{F}_{u,t}^T$

Correction

3. $\mathbf{z}^i_{exp,t} = h^i(\mathbf{x}'_t)$
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7. $\mathbf{x}_t = \mathbf{x}'_t + \mathbf{K}_t \mathbf{v}_t$
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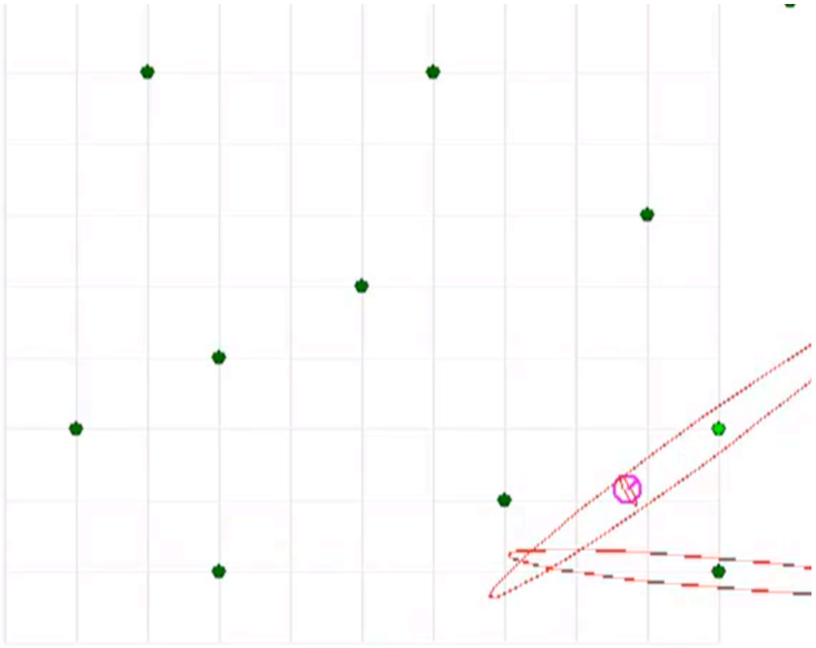
EKF SLAM



http://www.youtube.com/watch?v=vCVS9WAffi4

SLAM

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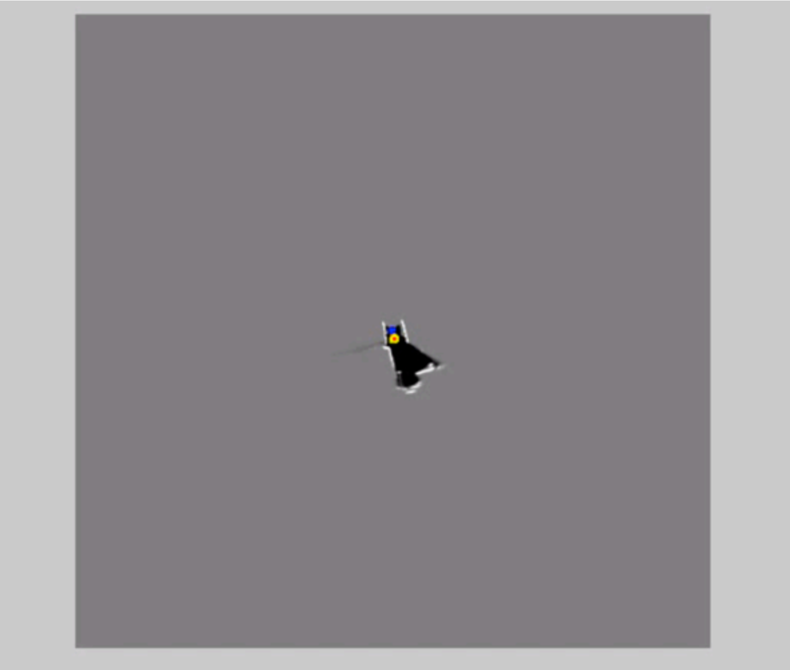
FastSLAM for Occupancy Grids



http://www.youtube.com/watch?v=1ENQtQ8nP3A