E190Q – Autonomous Mobile Robots

Lecture 11b

Kalman Filter Localization

INTRODUCTION
This problem set is meant to provide practice questions that aid students in learning Kalman Filter localization as well as provide

0. Single DOF KF
Given 2 consecutive range measurements of 2.50m, and 3.00m, fuse them using a KF. Assume the variance associated with the range measurements can be calculated as a function of range z, i.e. $\sigma^2 = 0.05z^2$.

1. Propagation of Errors A

A range measurement from the origin of an X-Y Cartesian coordinate frame to the center of an object is 5.00 meters and is taken at an angle of 30 degrees with respect to the X-axis of the coordinate frame. If the range and angle have respective variances of 2 degrees^2 and 0.01 m^2, what is the position estimate of the object with respect to the coordinate frame. What is the covariance matrix associated with this position estimate? Assume the range and angle measurements are independent.

2. Propagation of Errors B

A robot uses odometry to predict its position after 0.1 seconds of movement. During this time step, encoders measure each wheel's forward movement as $\mathbf{u}_t = [\Delta s_R \;\; \Delta s_L] = [0.010 \;\; 0.010]$ meters from previous position $\mathbf{x}_{t-1} = [x \; y \; \theta] = [1.00 \; 1.00 \; 0.00]$. It uses the motion model $\mathbf{x'}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_t)$ developed in class. The previous covariance matrix associated with the robot's mean state vector is:

$$\mathbf{P}_{t-1} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

What are $\mathbf{x'}_t$ and $\mathbf{P'}_t$? State any assumptions.

3. Confidence ellipse.

A mean state vector $\mathbf{x} = [\, x\ y\ \theta\,]^T$ and corresponding covariance matrix **P** characterize a 3DOF Gaussian distribution used to describe the belief of a robot's state. Sketch (roughly) the 2D position states confidence ellipse associated with each mean state vector and covariance matrix below.

a) $\mathbf{x} = [\, 2\ 2\ 0\,]^T$ $\quad\quad \mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b) $\mathbf{x} = [\, 2\ \text{-}2\ 0\,]^T$ $\quad\quad \mathbf{P} = \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c) $\mathbf{x} = [\, \text{-}2\ \text{-}2\ 0\,]^T$ $\quad\quad \mathbf{P} = \begin{bmatrix} 1 & -0.5 & 0 \\ -0.5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Innovation

Given the predicted state covariance matrix $\mathbf{P}_t$, the expected measurement function $\mathbf{z}_{exp,t} = h^i(\mathbf{x'}_t, \mathbf{M})$, and the measurement covariance matrix $\mathbf{R}_t$. of the actual measurement $\mathbf{z}_t$, calculate the covariance matrix associated with innovation. Be sure to specify the dimensions of the covariance matrix.

5. Review

Go through the KF algorithm slides. Make sure you understand where each step comes from.