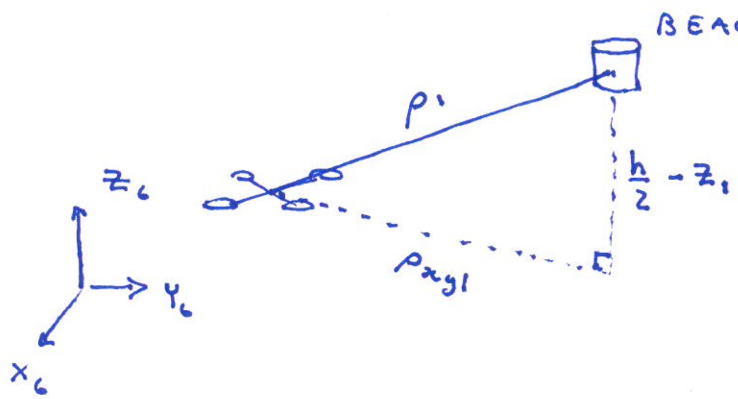
(1) a) ASSUME YAW ANGLE θ IS KNOWN

LET'S START WITH VERTICAL DIMENSION, USING PRESSURE SENSOR

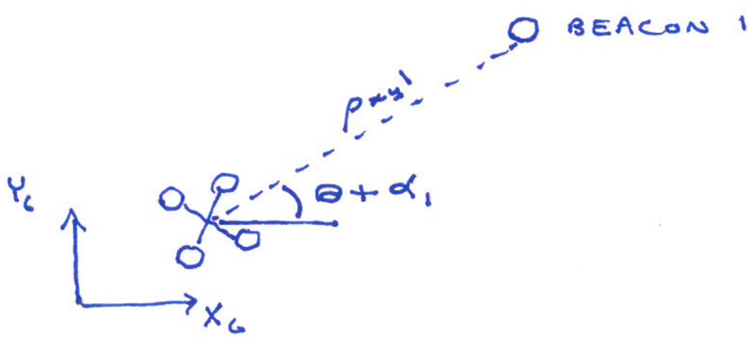
$$z_1 = a$$

NOW, THE RANGE TO BEACON IS BETWEEN TWO 3D POSITIONS, LET'S PROJECT INTO XY PLANE

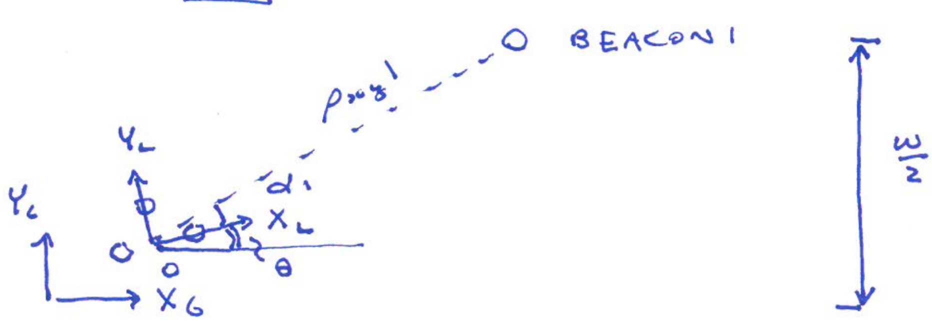


$$\rho_{xy1} = \sqrt{\rho_1^2 - \left(\frac{h}{2} - a\right)^2}$$

LETS DRAW A TOP DOWN VERSION WITH OUR PROJECTED RANGE



$\boxed{\text{OR}}$



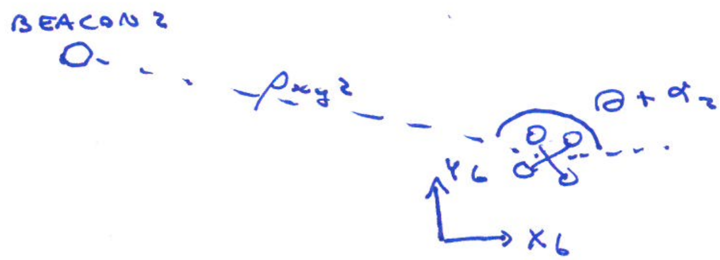
$$x_1 = \frac{l}{2} - \rho_{xy1}\cos(\theta+\alpha_1)$$

$$y_1 = \frac{w}{2} - \rho_{xy1}\sin(\theta+\alpha_1)$$

$$z_1 = a$$

5.5 marks

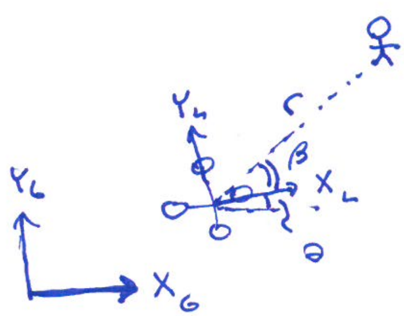
b) SAME EQ'NS AS ABOVE, BUT NEED TO WATCH SIGNS ON EQ'N TERMS



$$x_2 = -\frac{l}{2} - \rho_{xy1}\cos(\theta+\alpha_2)$$

1.5 marks

c) TOP DOWN VIEW



$$x_{in} = x_r + r\cos(\theta+\beta)$$

$$y_{in} = y_r + r\sin(\theta+\beta)$$

$$z_{in} = z_r$$

② a) USE 12DOF VECTOR SINCE $f(\,)$ NEEDS IT

$$X = \begin{bmatrix} x \\ y \\ z \\ \gamma \\ \psi \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\gamma} \\ \dot{\psi} \\ \dot{\theta} \end{bmatrix} \qquad P = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \cdots & \sigma_{x\dot{\theta}} \\ \sigma_{xy} & \sigma_{yy} & & \\ \sigma_{xz} & & \ddots & \\ \vdots & & & \\ \sigma_{\dot{\theta}x} & & & \sigma_{\dot{\theta}\dot{\theta}} \end{bmatrix}$$

b) TO PREDICT STATE, USE GIVEN DYNAMIC MODEL $f(\,)$

$$X_t' = f(X_{t-1}, U_t, \Delta t)$$

TO PREDICT COVARIANCE

$$P_t' = F_X P_{t-1} F_X^T + F_u \Sigma_u F_u^T$$

WHERE $F_X = \left[\dfrac{\partial f}{\partial X_{t-1}}\right] = \begin{bmatrix} \partial x_t/\partial x_{t-1} & \partial x_t/\partial y_{t-1} & \partial x_t/\partial z_{t-1} & \cdots \\ \partial y_t/\partial x_{t-1} & \partial y_t/\partial y_{t-1} & \partial y_t/\partial z_{t-1} & \\ \vdots & & & \\ & & & \end{bmatrix}$

$$F_u = \left[\frac{\partial f}{\partial U}\right] = \begin{bmatrix} \partial x_t/\partial u_1 & \partial x_t/\partial u_2 & \partial x_t/\partial u_3 & \partial x_t/\partial u_4 \\ \partial y_t/\partial u_1 & & \ddots & \\ \vdots & & & \end{bmatrix}$$

ASSUME $\Sigma_u = \begin{bmatrix} k|u_1| & & & \\ & k|u_2| & 0 & \\ & 0 & k|u_3| & \\ & & & k|u_4| \end{bmatrix}$

c) LET THE INNOVATION BE A DIFFERENCE BETWEEN BEACON MEASUREMENTS OF POSITION AND PREDICTED POSITION

$$v = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \\ a \end{bmatrix} - \begin{bmatrix} x' \\ y' \\ x' \\ y' \\ x' \\ y' \\ x' \\ y' \\ z' \end{bmatrix}$$

$\uparrow$ MEASURED $\qquad$ $\uparrow$ EXPECTED = PREDICTED STATES

$$\Sigma_{in} = G R_t G^T + H P_t' H^T$$

$$G = \left[\partial g / \partial \alpha_i \beta_i \right] = \begin{bmatrix} \partial x_1/\partial \rho_1 & \partial x_1/\partial \alpha_1 & \partial x_1/\partial \rho_2 & \cdots \\ \partial y_1/\partial \rho_1 & \partial y_1/\partial \alpha_1 & & \\ \partial x_2/\partial \rho_1 & & & \\ \vdots & & & \end{bmatrix}$$

$$R = \begin{bmatrix} \sigma_{\rho_1}^2 & & & 0 \\ & \sigma_{\alpha_1}^2 & & \\ & & \sigma_{\rho_2}^2 & \\ 0 & & & \ddots \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & 0 & & 0 \\ 0 & 1 & 0 & 0 & & 0 \\ & \vdots & & & & \\ 0 & 0 & 0 & 0 & & 1 \end{bmatrix}$$

$$K_t = P_t' H_t^T (\Sigma_{IN})^{-1}$$

$$X_t = X_t' + K_t v_t$$

$$P_t = P_t' - K_t \Sigma_{IN} K_t^T$$

d) BOTH THE STATE VECTOR AND COV. MATRIX MUST BE EXTENDED

$$X_t = \begin{bmatrix} x \\ y \\ z \\ \gamma \\ \psi \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\gamma} \\ \dot{\psi} \\ \dot{\theta} \\ x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ x_3 \\ y_3 \\ z_3 \\ x_4 \\ y_4 \\ z_4 \end{bmatrix} \quad P_t = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \cdots & \sigma_{xz_4} \\ \sigma_{yx} & \sigma_{yy} & & \\ \vdots & & \ddots & \\ \sigma_{z_4 x} & & & \sigma_{z_4 z_4} \end{bmatrix}$$

③ a) $p = \begin{bmatrix} x \\ y \\ z \\ \gamma \\ \psi \\ \theta \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\gamma} \\ \dot{\psi} \\ \dot{\theta} \\ w \end{bmatrix}$ ← weight

b) FOR particles $i = 1 \dots N$

$\Delta t_i =$ ~~[illegible]~~ SIZE OF PF TIME STEP

$$U_i = U_t + \begin{bmatrix} nrand(0, \sigma_u) \\ nrand(0, \sigma_u) \\ nrand(0, \sigma_u) \\ nrand(0, \sigma_u) \end{bmatrix}$$

normal dist.

0 mean

σ_u = experimentally determined process noise associated with control inputs to model

$$x_{i_t} = f(x_{i_{t-1}}, U_i, \Delta t_i)$$

c) for particles $i = 1 \dots N$

$$w_i = \exp\left(\frac{-v^T v}{\sigma_v} \right)$$

← v from KF

$\sigma_v = \|\Sigma_{in}\|$ from KF

$= \sqrt{\sigma_{v_1}^2 + \sigma_{v_2}^2 + \dots \sigma_{v_q}^2}$

(ASSUME INNOVATION IS A NORMALLY DISTRIBUTED VAR.)

THEN RESAMPLE WITH ONE OF METHODS FROM CLASS

(4)

$$\Delta t_r = \text{rand}()[0.4] + 0.1$$

↑

PICK random time step from 0.1 to 0.5 s

$$U_{n'} = \begin{bmatrix} \text{rand}()[u_{max} - u_{min}] + u_{min} \\ \vdots \\ \text{rand}()[u_{max} - u_{min}] + u_{min} \end{bmatrix}$$

↑ max control input possible

↑ min control input possible

$$X_{n'} = f(X_n, U_{n'}, \Delta t_r)$$

NOW USE ITERATIVE COLLISION CHECK:

$d_{sum} = 0$

$d_{n \to n'}$ = dist from n to n'

$X_{n^*} = X_n$

WHILE $(d_{sum} < d_{n \to n'})$

 d = dist from X_{n^*} to closest obstacle

 $d_{sum} \mathrel{+}= d; \quad \Delta t^* = \frac{d_{sum}}{d_{n \to n'}} \Delta t_r$

 $X_{n^*} = f(X_n, U_{n'}, \Delta t^*)$

 IF $(d < d_{min})$

  return COLLISION

RETURN NO COLLISION

(5)

1 a
2 c
3 a
4 d
5 d
6 b