

# E160 - Autonomous Robot Navigation

## Midterm Exam

March 10, 2016

Name:

Signature:

**Time Limit: 1.5 continuous hours, submit by 9am Friday March 11<sup>th</sup> unless other accommodations have been made with the instructor**

**Open Book**

**Open Notes**

**Open Computers – Matlab, Excel can be used, E160 website only version of internet use**

### 0. INTRODUCTION

Consider a quadrotor used for security in a warehouse (see Fig. 1a). The robot has 4 *rotors*, one placed on each side of the robot. Assume that each of the 4 rotors can be actuated independently with control signal  $u_i$ ,  $i=1..4$ , to provide control over all 6 degrees of freedom. Note the X, Y, Z axes of the robot's local coordinate frame are shown.

The quadrotor will operate in a large rectangular warehouse of dimensions  $l \times w \times h$ , (Fig 1b). Located at the four corners of the warehouse ceiling, are uniquely identifiable beacons, (see colored cylinders in Fig. 2b). The quadrotor is equipped with an upward facing omnidirectional camera that can detect each of the four beacons at all times, and measure the relative angle  $\alpha_j$  and distance  $\rho_j$  to the  $j^{\text{th}}$  beacon, where  $j = 1..4$ . Note that angle  $\alpha_j$  is taken about the  $Z_L$  axis, and  $\alpha_j = 0$  rad when the  $X_L$  axis is pointed directly at the  $i^{\text{th}}$  beacon. The distance  $\rho_i$  is the distance taken between the 3D position of the quadrotor and the 3D position of the  $i^{\text{th}}$  beacon. The quadrotor also has a pressure sensor that can be used to measure altitude  $a$ , that is, it measures the height of the quadrotor along the  $Z_G$  axis.

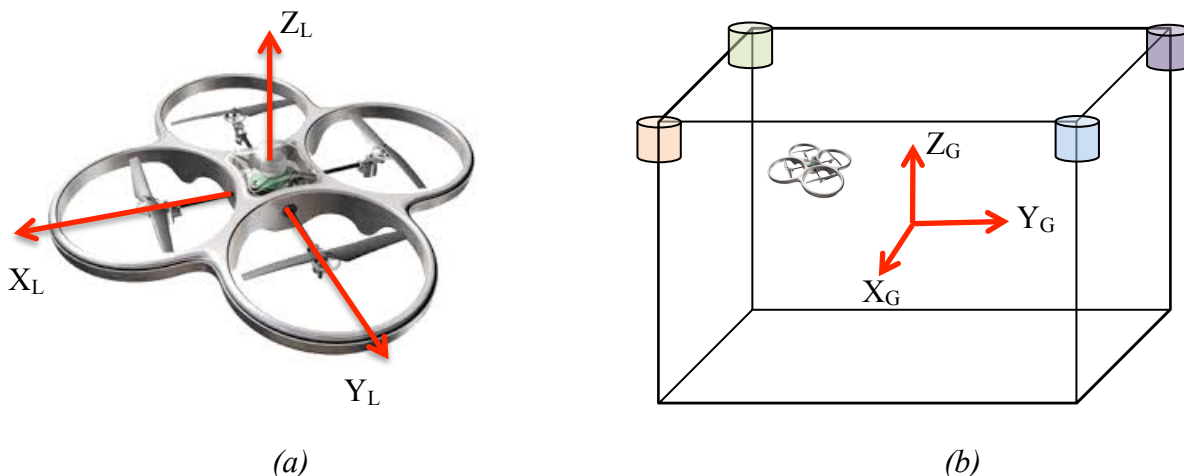


Figure 1: A quadrotor is shown in (a). In (b) a warehouse is shown with beacons on the ceiling.

Assume that the bearing and range beacon measurements are normally distributed random variables with standard deviations  $\sigma_a$  and  $\sigma_p$  respectively. The altitude measurements are characterized similar to the bearing measurements, (i.e. normally distributed), but with standard deviation  $\sigma_a$ .

The discrete time dynamic model of the vehicle can be expressed generally as:

$$X_t = f(X_{t-1}, U_t, \Delta t)$$

Where  $X_t$  is a 12 DOF state vector containing 3 position states and 3 angle states, as well as their 6 associated velocity states. The vector  $U = [u_1 \ u_2 \ u_3 \ u_4]$  is the controllable rotor speed vector. The size of the time step is  $\Delta t$  seconds.

### 1. SENSING (10 points)

For question 1, assume roll and pitch angles are 0 rad.

- a) Provide an equation that describes the position of the robot with respect to the global coordinate frame in terms of measurements to beacon 1. Call this measurement  $[x_l \ y_l \ z_l]$ . Assume the global coordinate frame is anchored to the field with its origin at the middle of the field and X axis parallel to the short side of the field. Draw a picture that includes the coordinate frames, a robot, and any variables.

- b) Write the same equations for measurements to beacon 2, 3, and 4.
- c) If the robot also has an on-board camera facing forwards to track an intruder that is at the same altitude as the quadrotor, provide an equation that describes the intruder's position  $[x_{In} \ y_{In} \ z_{In}]$  in the global coordinate frame as a function of the robot's position and the on-board camera's measurement  $[r \ \beta]$ , where  $r$  is the range to the intruder, and  $\beta$  is the relative bearing angle to the intruder, taken about the  $Z_L$  axis.

**2: KF LOCALIZATION (15 points)**

Design a Kalman Filter (KF) that will localize the quadrotor. In this case assume the roll, pitch, angles are both zero.

a) What variables should be in the State Vector? In the Covariance Matrix?

b) Describe a prediction step of the KF algorithm that updates the state vector and associated covariance matrix. Assume there are no proprioceptive sensors. State any other assumptions.

c) Describe a correction step for the KF. Be sure to define the innovation vector, the update equation for the innovation covariance, the state vector update and the covariance matrix update. State any assumptions.

d) How can the KF localization algorithm you designed be extended to run SLAM and estimate the position states of the beacons?





#### 4: MOTION PLANNING (5 points)

A single query Probabilistic Road Map (PRM) will be used to plan paths for the robot.

- a) Propose a node expansion step. Specifically, given a node  $n$  in the roadmap defined by the configuration  $X_n = [x_n \ y_n \ z_n \ \alpha_n \ \beta_n \ \gamma_n \ \dot{x}_n \ \dot{y}_n \ \dot{z}_n \ \dot{\alpha}_n \ \dot{\beta}_n \ \dot{\gamma}_n]$  that has been selected for expansion, provide the equations that describe the configuration  $X_{n'}$  of a newly generated node  $n'$ . To note, in this search, the first three states describe position, and the next three states are Euler angles roll, pitch and yaw. The final six states are velocities.

- b) Define an endgame region for such a planner.



**5: MULTIPLE CHOICE – 2 points awarded for each correct answer, 0 points if incorrect or no answer. Circle the answer that BEST completes the sentence.**

1. The confidence ellipse is typically drawn to represent
  - a) a 2D normal distribution
  - b) a particle set
  - c) uncertainty in a sensor
  - d) a Markov discretization.
  
2. The denominator in the Baye's rule equation for Markov localization
  - a) is typically calculated using extensive sensor modeling.
  - b) can only be calculated assuming the state is normally distributed.
  - c) is calculated by leveraging knowledge that the likelihood of all possibilities equals 1.
  - d) None of the above.
  
3. A Kalman Filter solution that fuses multiple measurements to calculate an estimate,
  - a) can be derived from a weighted least squares solution.
  - b) uses linearization techniques via Jacobian matrices.
  - c) assumes sensor measurements are modeled as bimodal Gaussian functions.
  - d) None of the above.
  
4. RRTs
  - a) Are slower than single-query PRMs in general
  - b) Suffer from clustering
  - c) Construct optimal paths
  - d) None of the above.
  
5. The ZLOG method,
  - a) detects zero crossings.
  - b) approximates the Laplacian function.
  - c) uses a Gaussian operator to deal with noise.
  - d) All of the above.

BONUS: The robot built by Kagome,

- a) uses wheeled locomotion.
- b) uses bipedal locomotion.
- c) is built as an autonomous mobile platform for perimeter security applications.
- d) All of the above.