E160 – Autonomous Robot Navigation

Exam Solutions 2018

1. a)

Assuming we know the mass, buoyancy and can accurately measure w, we set u_V according to

$u_V = 1/C_V\ [F_{drag}w+mg-F_B+K(w_{des} - w)]$

2 points – having the P control term $K(w_{des} - w)]$
2 points – having the other terms that will cancel out the gravity, buoyancy, drag.

b)

Substituting uV into the V term of the dynamics equation for rotation yields:

w_dot = $K(w_{des} - w)]$

defining error as

$e = w_{des} – w$

Now, noting that w w_{des}_dot = 0, we get

e_dot = w_{des}_dot - w_dot
= 0 – Ke

so
e_dot = -Ke

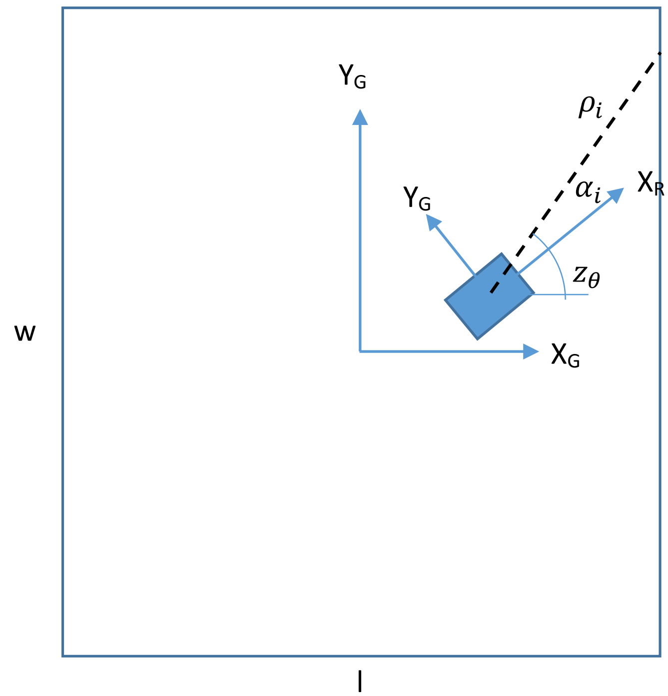
This will drive the error to zero, or let us attain a desired rotational velocity if $K > 0$.

2 points – substituting in the control
2 points – obtaining an equation of the form e_dot = Ae
2 points – selecting an appropriate control gain for stability

2. a)

$x_i = \frac{l}{2} - \rho_i \cos(z_\theta + \alpha_i)$

y_i can not really be determined from this particular measurement



2 points for x_i

2 points for y_i

1 point for diagram having global coordinate frame and local coordinate frame

1 point for lines being drawn appropriately.

2 point for labeling all variables

b)

Since the sonar beam is conical and not linear, there will be reflections from the bottom of the tank which may mislead the algorithm into thinking that a wall is closer than it is.

2 points

3. a)
State is 1x7 or 7x1
$X = [x\ y\ \theta\ \dot{x}\ \dot{y}\ \dot{\theta}\ z]$

Covariance is 7x7

$$P = \begin{bmatrix} \sigma_{xx} & \sigma_{yx} & \cdots \\ \sigma_{xy} & \sigma_{yy} & \cdots \\ \cdots & \cdots & \sigma_{\dot{\theta}\dot{\theta}} \end{bmatrix}$$

2 points – state vector and covariance matrix

b)
Predicting the horizontal states with the dynamics equations:
$X_t{}' = f(X_{t-1}, \boldsymbol{u}_t, \Delta t)$

Predicting the vertical states with the dynamics equations:
$z_t{}' = g(z_{t-1}, \boldsymbol{u}_t, \Delta t)$

Predict the covariance using previous covariance matrix P_{t-1}, and the covariance matrix $Q_{u,t}$ associated with the uncertainty in control thrust effects. Specifically, we will assume the Vertical, Right, and Left control thrusts will have independent variance terms:

$$Q_{u,t} = \begin{bmatrix} \sigma_{uV}^2 & 0 & 0 \\ 0 & \sigma_{uR}^2 & 0 \\ 0 & 0 & \sigma_{uL}^2 \end{bmatrix}$$

Now the predicted covariance Matrix is:
$P_t' = F_{X,t} P_{t-1} F_{X,t}^T + F_{u,t} Q_{u,t} F_{u,t}^T$

In this case $F_{X,t}$ is the Jacobian that relates previous states to current states and will contain terms like $\frac{dx_t{}'}{dy_{t-1}}$ and $\frac{dz_t{}'}{dx_{t-1}}$.

Similarly, $F_{u,t}$ is the Jacobian that relates the input thrust values to current states and will contain terms like $\frac{dx_t{}'}{du_{R,t}}$ and $\frac{dz_t{}'}{du_{V,t}}$.

1 point – state vector update equations
1 point – covariance matrix update equation
1 point – explanation of Jacobians

c)
The innovation in this case is
$v_t = \mathbf{Z_t} - \mathbf{Z}_{exp,t}$

Where $\mathbf{Z_t}$ is the vector of measurements extracted in questions 2. a)
$\mathbf{Z_t} = [x_1\ x_2\ y_3\ y_4\ ...\ x_N]$

Note that in the measurement vector above, each measurement *i* only yields an x_i OR y_i value, not both. Depending on the time step, and which wall a laser beam is hitting, you

The expected measurement then, will just be the predicted x or y value.
$\mathbf{Z_{exp,t}} = [x_t{}'\ x_t{}'\ y_t{}'\ y_t{}'\ ...\ x_t{}']$

The innovation's covariance can then be described by:
$\Sigma_{IN,t} = H_{x',t} P'_t H^T_{x',t} + L_{Z,t} R'_t L^T_{Z,t}$

Where $H_{x',t}$ is the Jacobian is that relates previous states to the innovation vector and will contain terms like $\frac{dv_1}{dy_t{}'}$ and $\frac{dv_3}{dy_t{}'}$. Note that $\mathbf{Z}_{exp,t}$ is a function of $x_t{}'$ and $y_t{}'$, while $\mathbf{Z}_t$ is a function of $\theta_t{}'$.

Also, $L_{Z,t}$ is the Jacobian is that relates range, bearing measurements to the innovation vector and will contain terms like $\frac{dv_1}{d\rho_2}$ and $\frac{dv_3}{d\alpha_1}$, and v_3 is the 3rd element of the innovation vector.

Also, R_t is the NxN covariance matrix that addresses the (independent) uncertainty in raw range/bearing sensor measurements.

$$R_t = \begin{bmatrix} \sigma^2_{\rho 1} & 0 & 0 \\ 0 & \sigma^2_{\alpha 1} & 0 \\ 0 & 0 & ... \end{bmatrix}$$

The Kalman gain can be calculated as:
$K_t = P'_t H^T_{x\prime,t} \left(\Sigma_{IN,t}\right)^{-1}$

The final updates of the state and covariance are:
$X_t = X'_t + K_t v_t$
$P_t = P'_t - K_t \Sigma_{IN,t} K^T_t$

1 point – for an appropriate innovation vector
1 point – for an appropriate innovation covariance equation
1 point – for describing jacobians and introduction of uncertainty equations (e.g. H)
1 point - for having the last 3 equations the produce, K_t, X_t, P_t

d) Assuming it is known that there are 4 walls, they can be estimated by extending the state vector to include the end points of each wall, or mid points, or slopes and intercepts, etc.. There are many possibilities here depending on assumptions

1 point – extending the state vector and covariance matrix

4.a)
a particle should include X_t and weight w.

1 point – state
1 point - weight

b)
Iterate on all particles, the propagation step and the weight calculation:

For propagation, use dynamic models (with randomness) to update particle position. E.g. for the ith particle:

$$X_t^i = f(X_{t-1}^i, \boldsymbol{u}_t^i, \Delta t)$$
$$\boldsymbol{u}_t^i = [u_{V,t} + randn(0, \sigma_{uV}) \quad u_{R,t} + randn(0, \sigma_{uR}) \quad u_{L,t} + randn(0, \sigma_{uL})\ ]$$

where randn(μ, σ) is a function that samples from a normal distribution of mean μ and standard deviation σ.

Calculate weight for the ith particle:

$$w_t^i = p(X_t^i, \boldsymbol{Z}_t)$$

where $\boldsymbol{Z}_t$ is the most recent measurement vector as defined in problem 3c) and p() is the weight calculation function

1-point iterate
1-point propagation equation
1-point add randomness
1-point calculate weight for each particle

c)
An exact or approximate algorithm can be used to randomly select a new set of *N* particles by drawing from the old set of *N* particles, while ensuring the likelihood of selection of the particles be proportional to the calculated weight of the particle.

If not described above in b), the weight function p can be set according to a variety of functions, (that should really be determined experimentally), including:

$$w_t^i = p\left(X_t^i, \boldsymbol{Z}_t\right) = \exp\left(-\frac{v_i^T v_i}{\sigma_v^2}\right) \exp\left(-\frac{(z - z^i)^2}{\sigma_z^2}\right) \exp\left(-\frac{(z_\theta - \theta^i)^2}{\sigma_\theta^2}\right)$$

In this case, v_i is the innovation vector from question 3 (without any heading or depth), and σ_v^2 is the covariance associated with range sensor measurements. The second term is similar but for depth. The third term is similar but for bearing.

2 points – description or pointer to resampling algorithm
2 points – weight calculation. Must include a difference between measurement and expected measurement.

5.
5.1 D
5.2 D
5.3 A
5.4 D
5.5 D
5.6 B