E160 – Lecture 15
Autonomous Robot Navigation

Instructor: Chris Clark

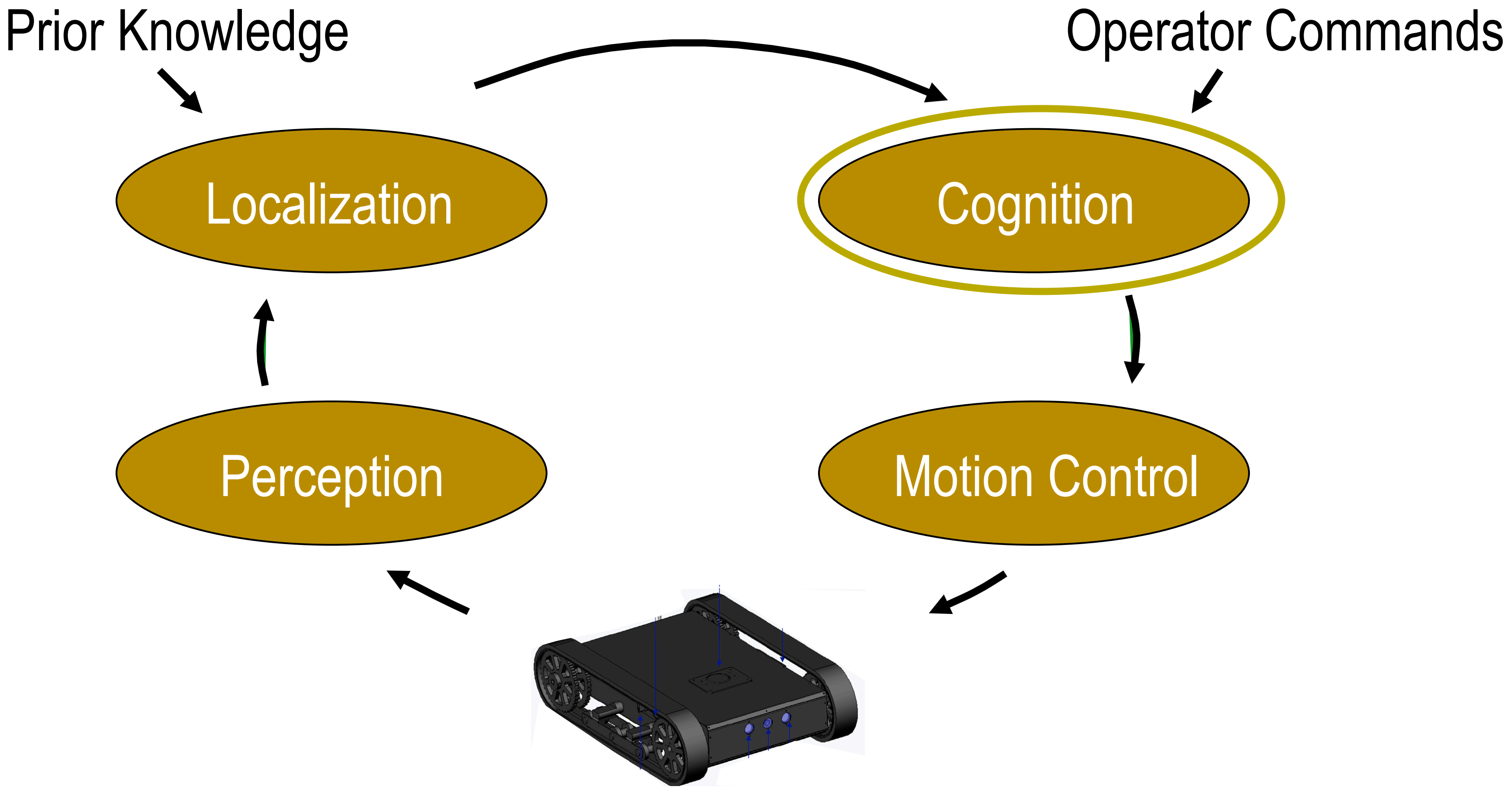
Semester: Spring 2016

Figures courtesy of Probabilistic Robotics (Thrun et. Al.)

MP: Outline

https://www.youtube.com/watch?v=Dw0WxPlyWIl

Control Structures
Planning Based Control



MP: Outline

1. Multi-Query PRMs
2. Graph Search
3. Artificial Potential Fields

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1. Multi-Query PRMs
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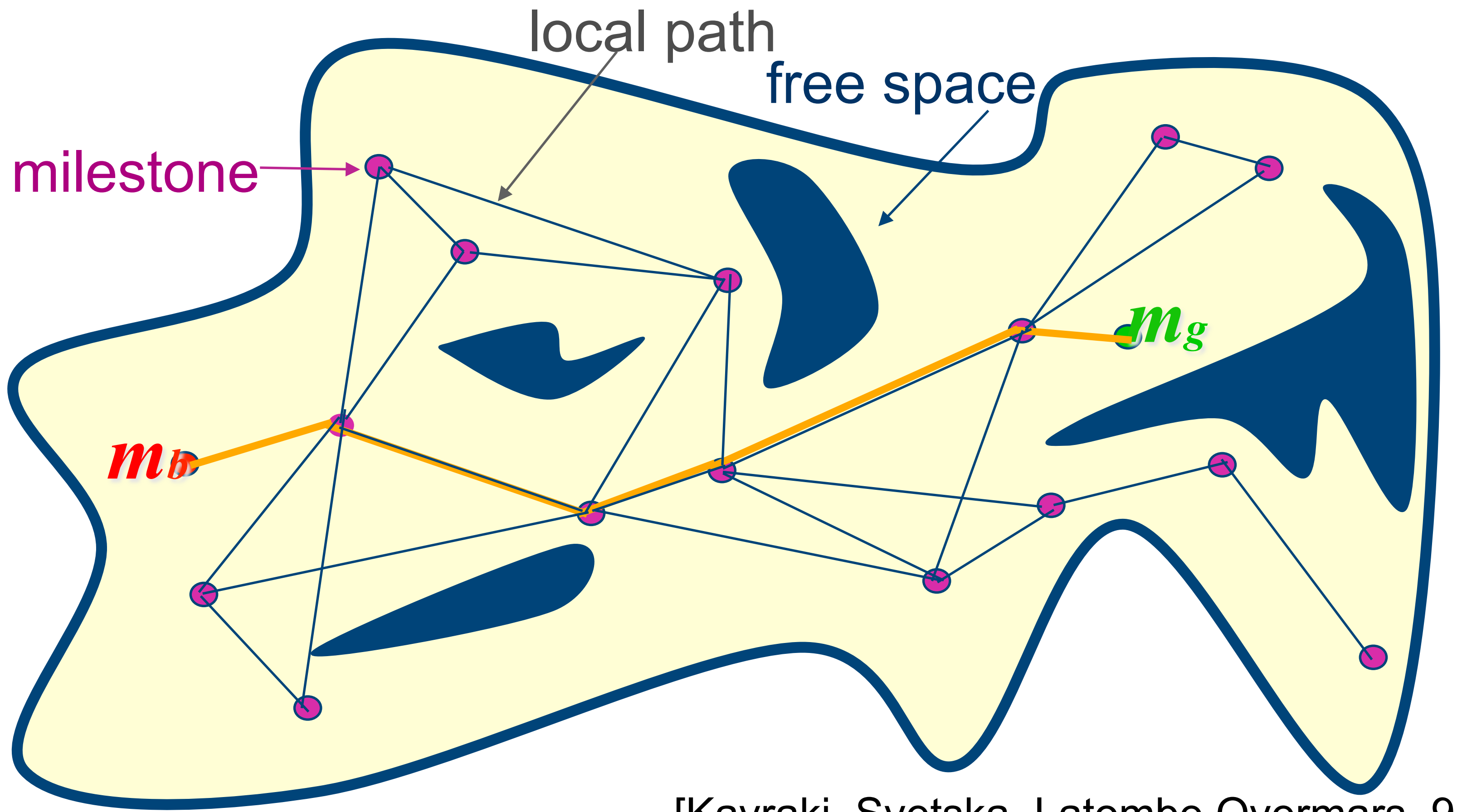
Multi-Query PRMs

- Multi-Query Strategy

1. Learning Phase:
   - Generate the PRM with two steps:
     - Construction
     - Expansion
2. Query Phase:
   - Connect start and goal configurations to PRM
   - Perform a graph search to find path

Multi-Query PRMs



[Kavraki, Svetska, Latombe,Overmars, 95]

Multi-Query PRMs

- Nomenclature

$R=(\mathbf{N}, \mathbf{E})$	RoadMap
$\mathbf{N}$	Set of Nodes
$\mathbf{E}$	Set of edges
c	Configuration
e	edge

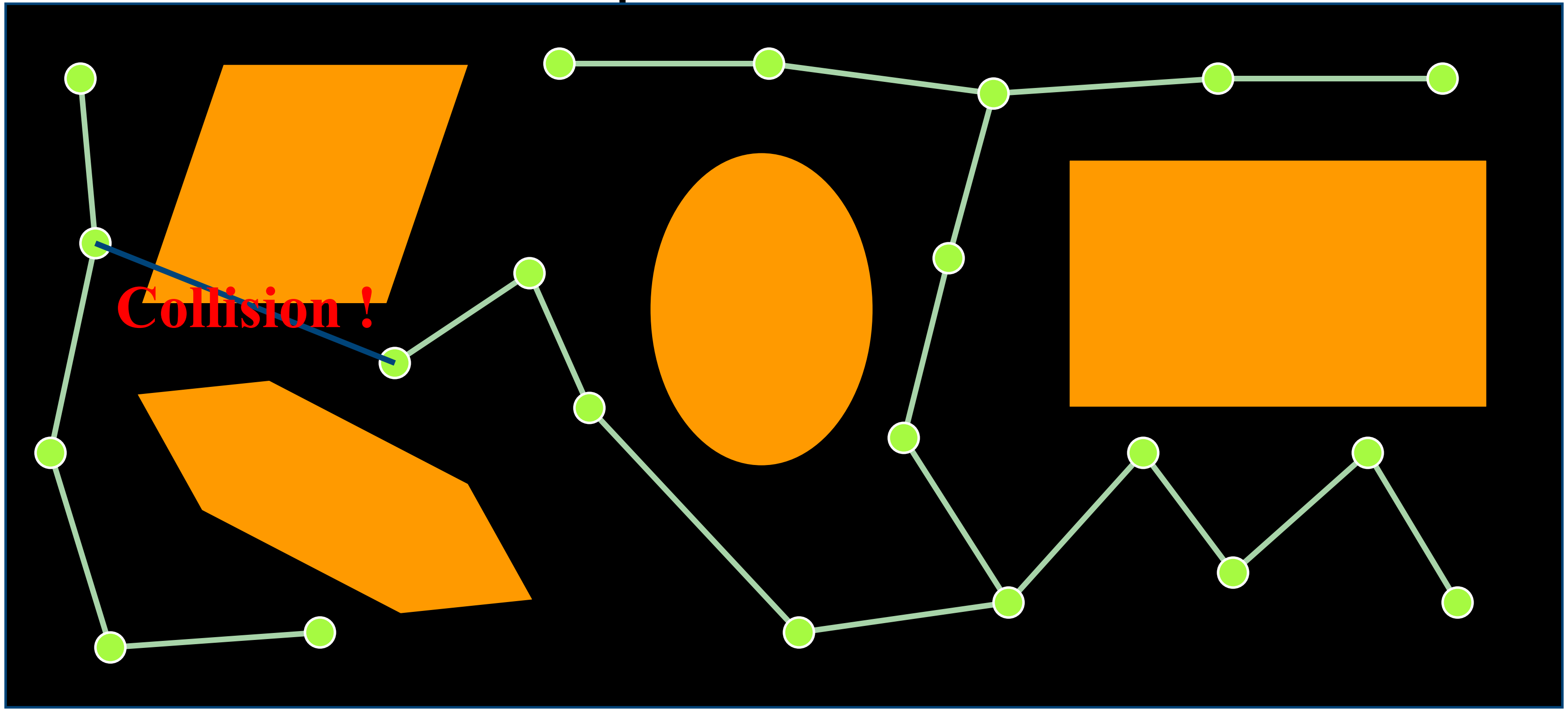
Multi-Query PRMs Learning Phase

- Construction Step Algorithm

```
Start with empty $R=(\mathbf{N}, \mathbf{E})$
while (not done)
{
    Generate a random free config $c$ and add to $\mathbf{N}$
    Choose a subset $\mathbf{N_c}$ of candidate neighbors around $c$ from $\mathbf{N}$
    Try to connect $c$ to each node in $\mathbf{N_c}$ with local planner in the
    order of increasing distance from $c$
    Add the edge found to $\mathbf{E}$
}
```

Multi-Query PRMs
Learning Phase

- Construction Step



Courtesy of C. Allocco

Multi-Query PRMs Learning Phase

- Local Planner
  - Used to connect two nodes.
  - Must contain collision-check.
  - For good performance, the LP must be:
    1. Deterministic - Eliminates the need for storing local plans.
    2. Fast - To ensure quick planning queries.

Multi-Query PRMs
Learning Phase

- Expansion Step
  1. Find the nodes in ‘difficult’ regions using <u>heuristic weight function</u> *w(c)*
  2. Expand *c* using <u>random-bounce walks</u>
  3. Repeat as necessary

Multi-Query PRMs
Learning Phase

- Expansion Step
  - Several options to define weight function $w(c)$
    - Inversely proportional to the "number of nodes within some predefined distance from c"
    - Inversely proportional to the "distance from c to the nearest connected component not containing c"
    - Proportional to the "failure ratio of the local planner"

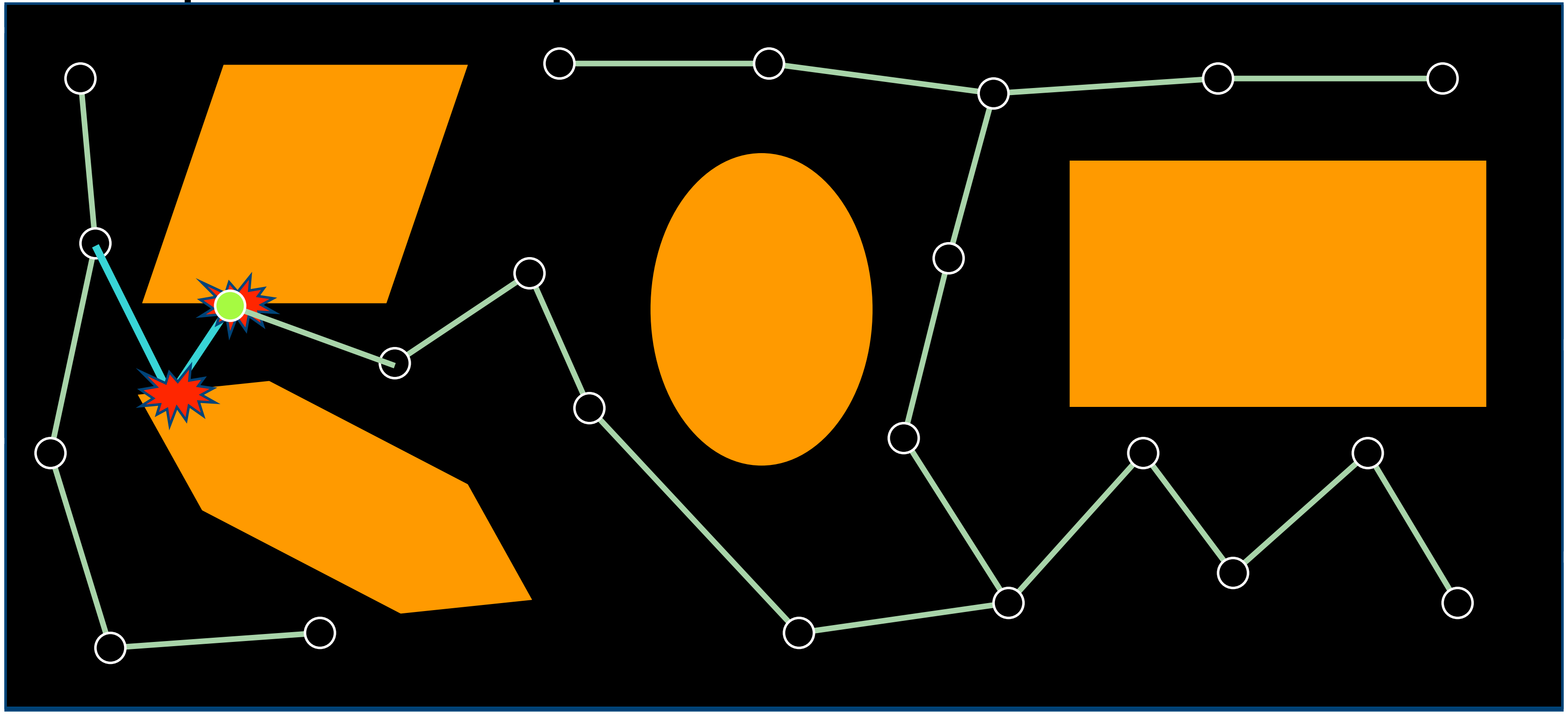
Multi-Query PRMs
Learning Phase

- Expansion Step

  1. Loop
     1. Pick a random direction of motion in C-space
     2. Move in the direction until an obstacle is hit
     3. Check for connection with another node
     4. Repeat until the path can be connected to another node

Multi-Query PRMs Learning Phase

- Expansion Step



Courtesy of C. Allocco

Multi-Query PRMs
Learning Phase

- Expansion Step

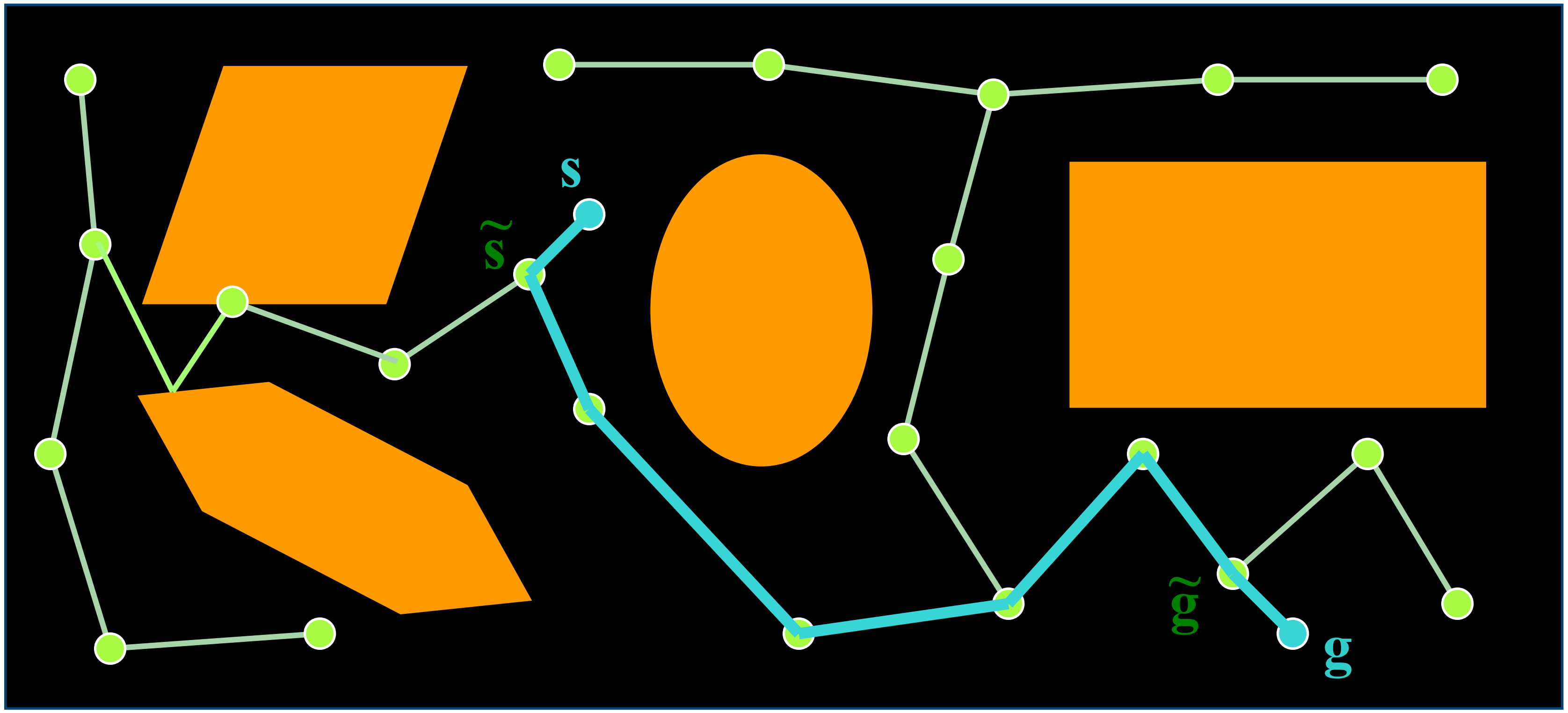
1. Loop
    1. Pick a random direction of motion in C-space
    2. Move in the direction until an obstacle is hit
    3. Check for connection with another node
    4. Repeat until the path can be connected to another node
2. Store the final config n and the edge (c, n) in R
3. Store the computed path (non-deterministic)
4. Record that n belongs to the same connected component as c

Multi-Query PRMs
Query Phase

- Query Phase Algorithm

1. Given the start and goal configurations s and g, calculate feasible paths P_s and P_g to the nodes $\tilde{s}$ and $\tilde{g}$ on the roadmap (w/ LP)
2. Calculate the path P from s to g using the roadmap and a **tree search** planner

Multi-Query PRMs
Query Phase



Courtesy of C. Allocco

Probabilistic Road Maps

- Two Tenets:
  1. Checking sampled configurations and connections between samples for collision can be done efficiently.
  2. A relatively small number of milestones and local paths are sufficient to capture the connectivity of the free space.

Probabilistic Road Maps: Discrete and Continous Planning

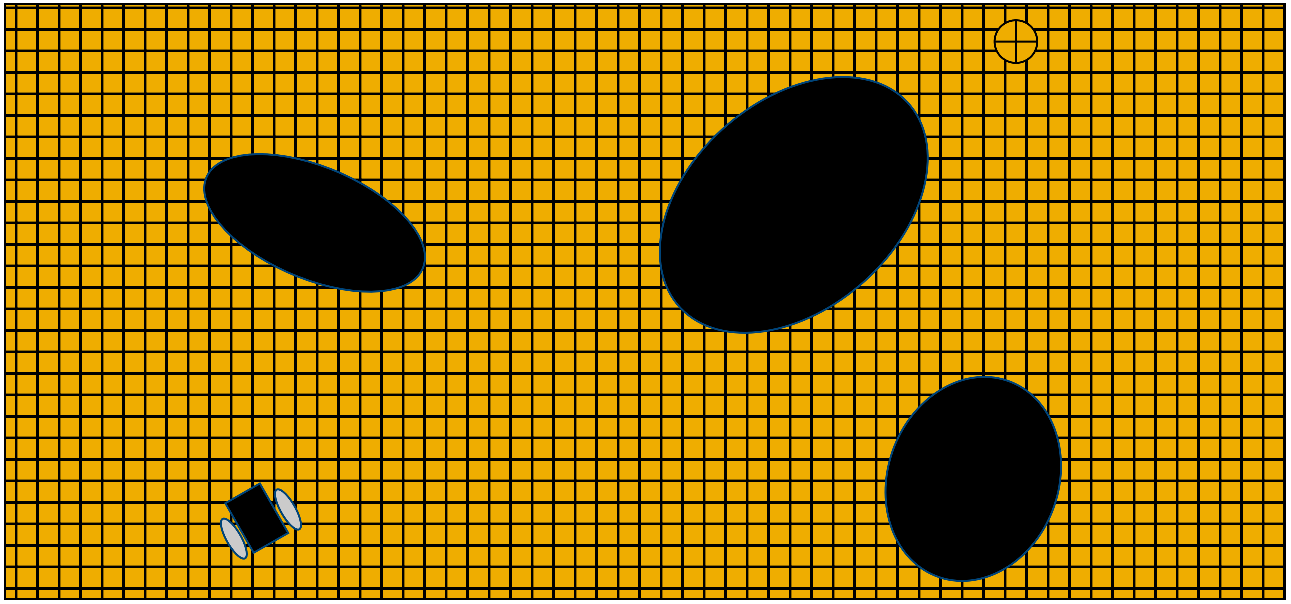
Courtesy of T. Bretl

MP: Outline

1. Multi-Query PRMs
2. Graph Search
3. Artificial Potential Fields

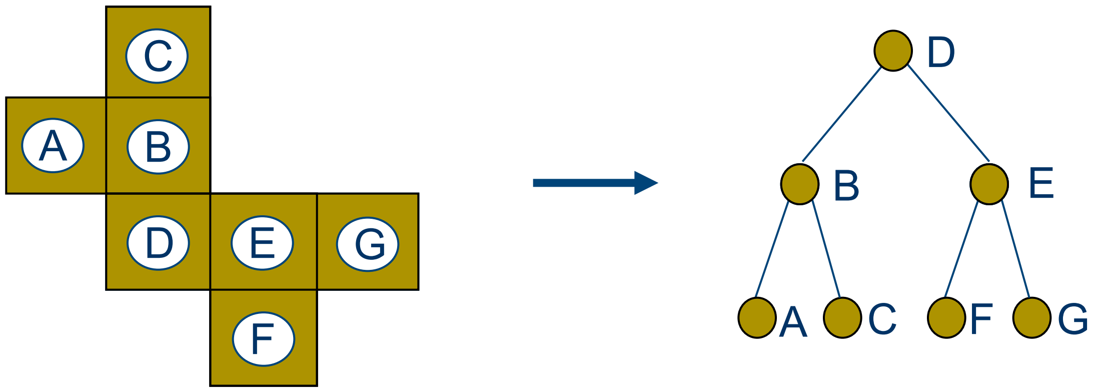
Graph Search

- Cell decomposition
  - Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells



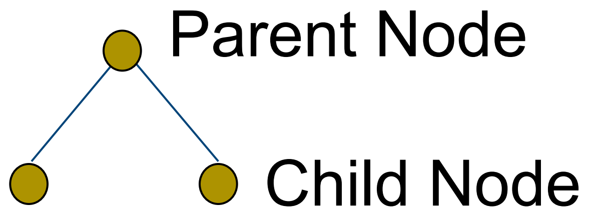
Graph Search

- Given a discretization of C, a search can be carried out using a Graph Search or gradient descent, etc.
  - Example: Find a path from D to G

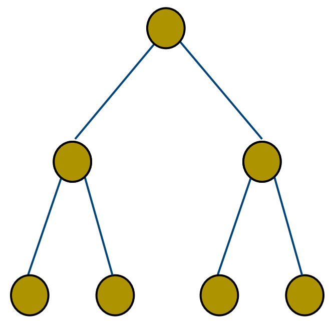


Tree Search

- Tree nomenclature:

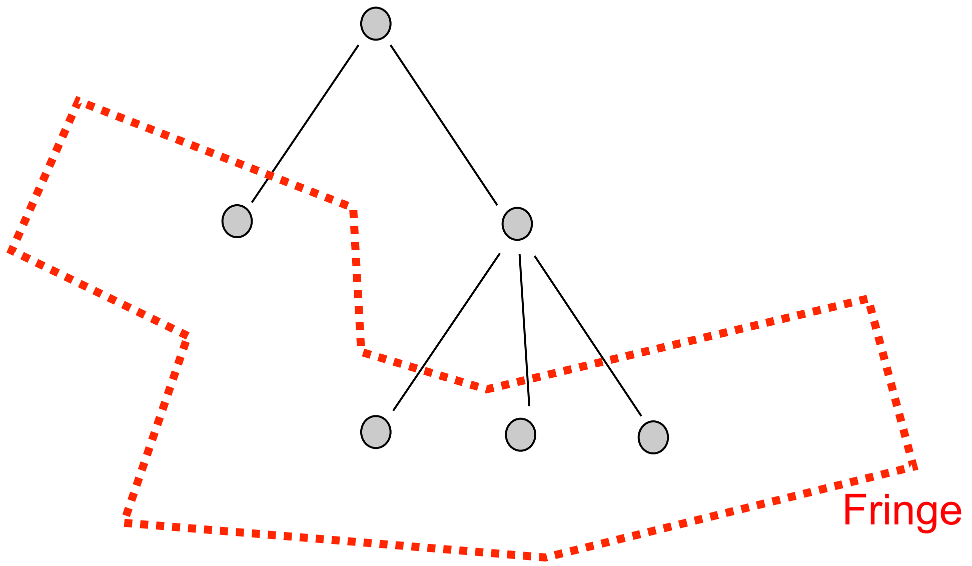


- Algorithms differ in the order in which they search the branches (edges) of the tree



Data Structures

- The **Fringe** or Frontier is the collection of nodes waiting to be expanded.



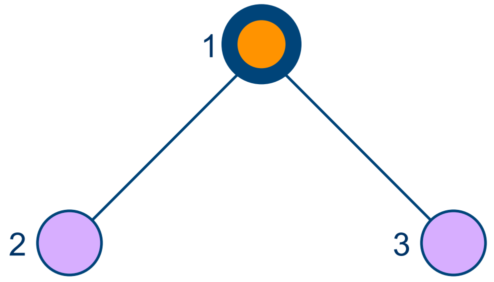
Tree Search

- Search Algorithms
  1. Breadth First Search
  2. Depth First Search
  3. A*

Breadth-First

- All the nodes at depth d in the search tree are expanded before nodes at depth $d+1$

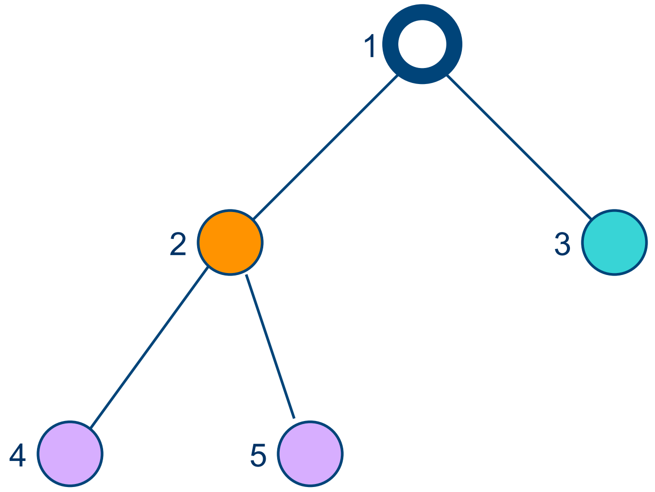
Breadth-First Snapshot 1



Initial 🔵
Visited ⚪
Fringe 🔵
Current 🟠
Visible 🟣
Goal 🔴

Fringe: [] + [2,3]

Breadth-First Snapshot 2

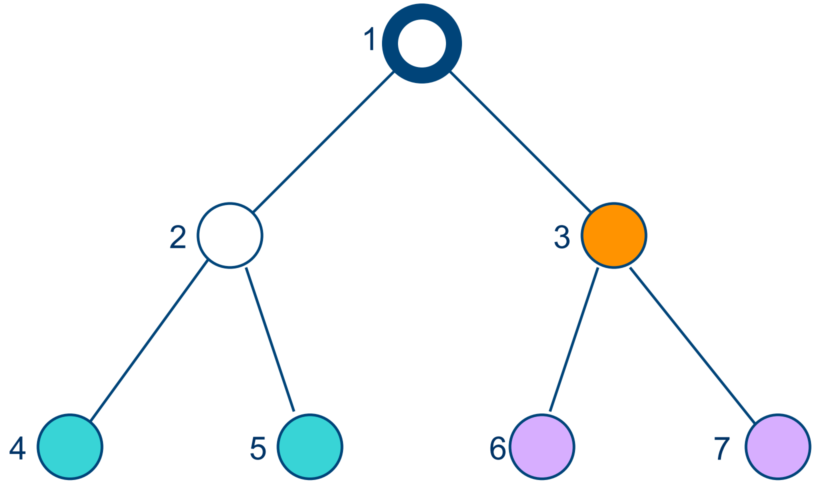


Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [3] + [4,5]

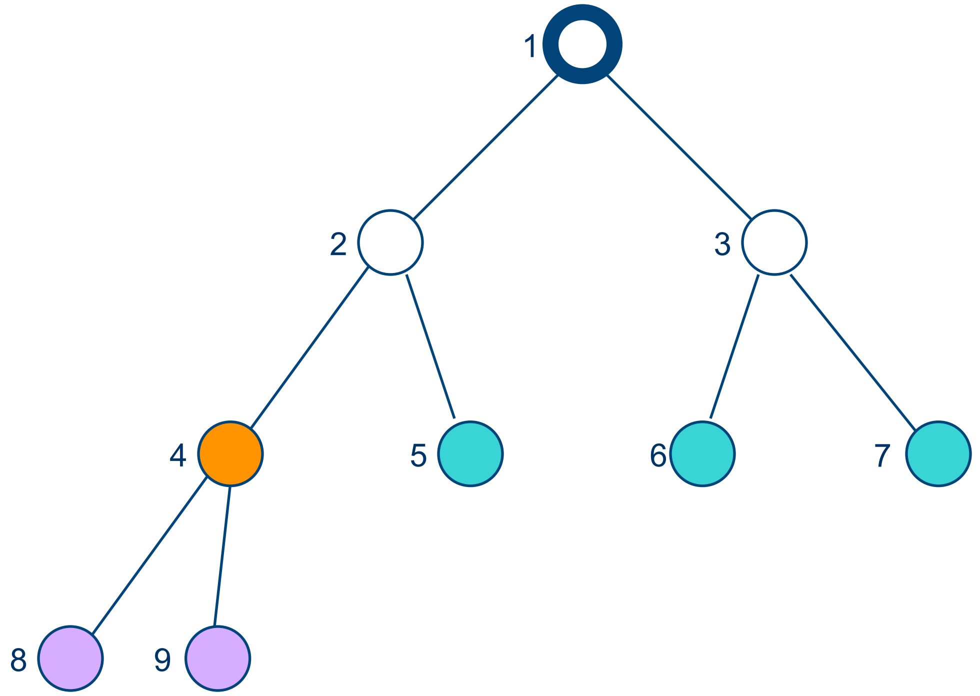
Breadth-First Snapshot 3

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●



Fringe: [4,5] + [6,7]

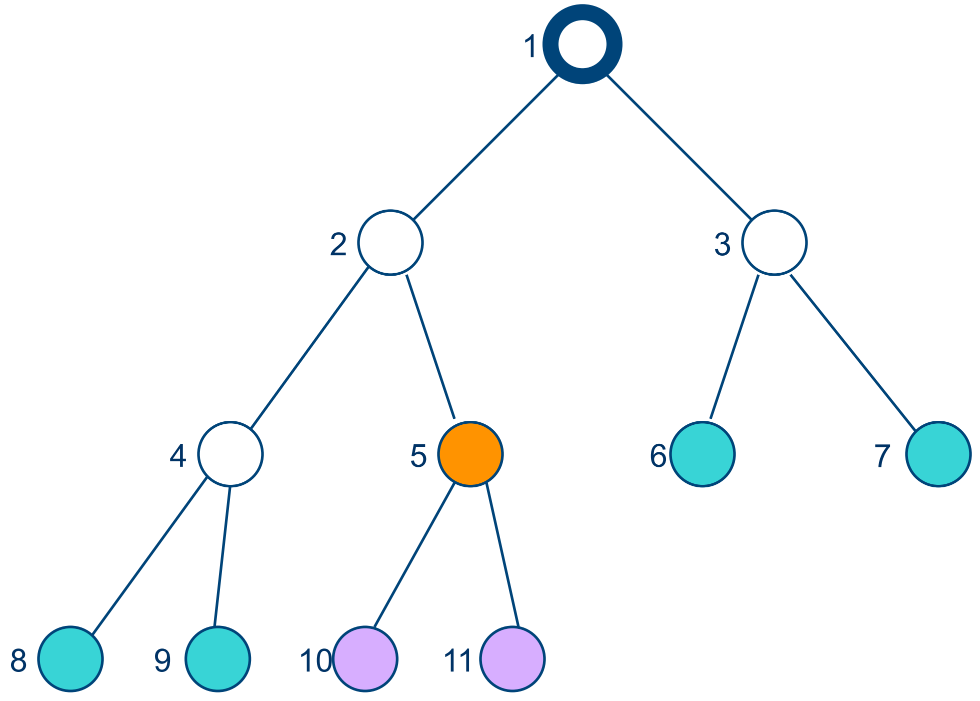
Breadth-First Snapshot 4



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [5,6,7] + [8,9]

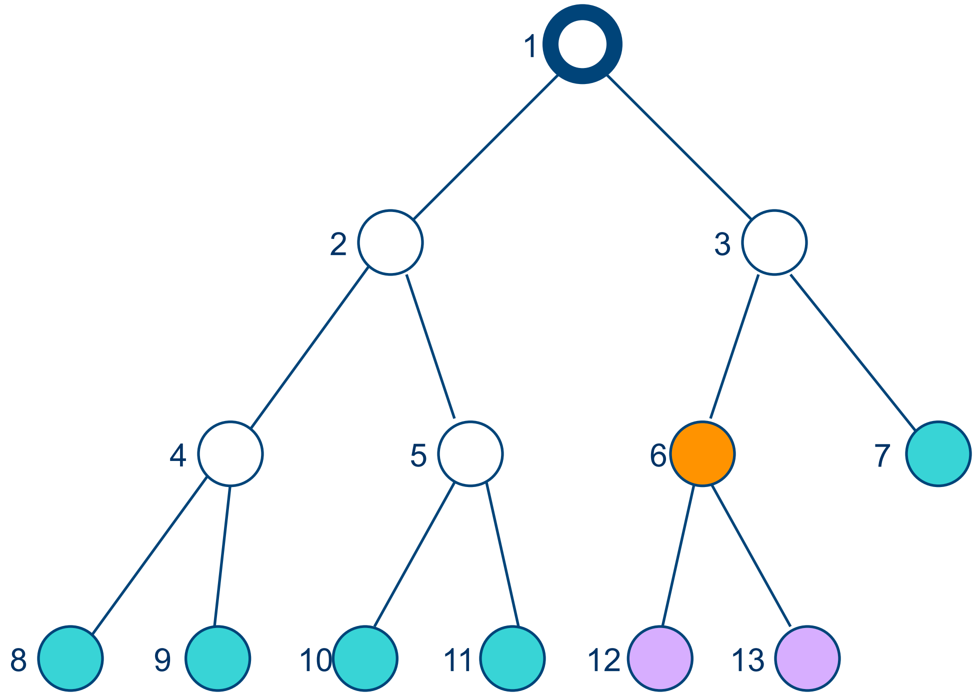
Breadth-First Snapshot 5



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [6,7,8,9] + [10,11]

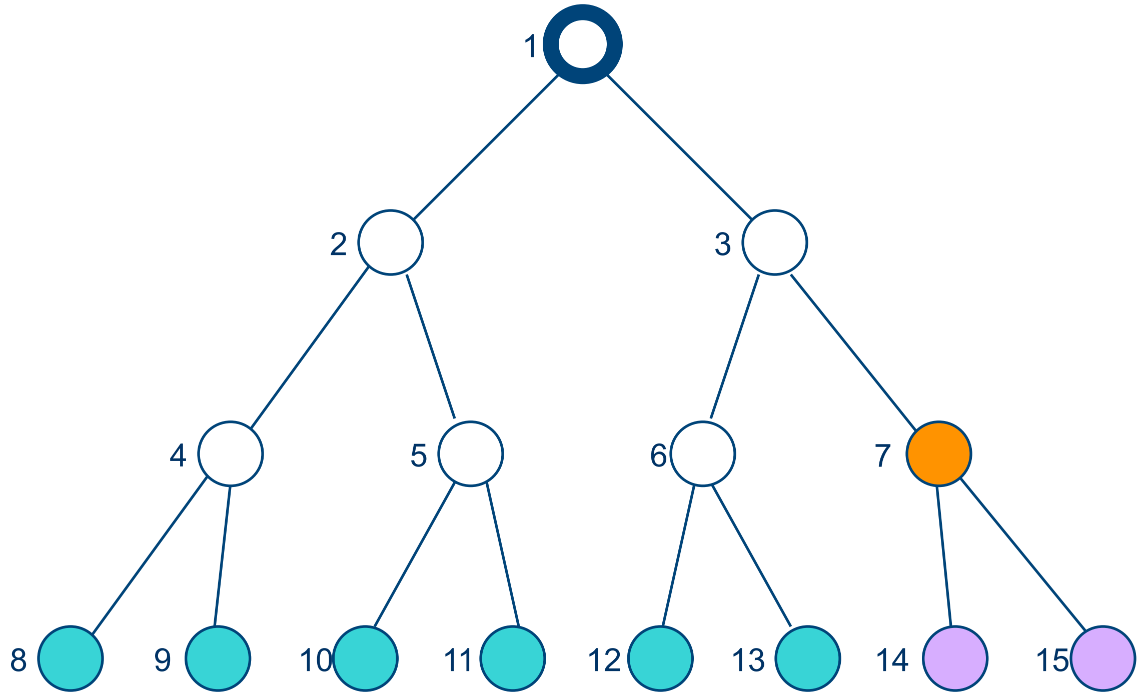
Breadth-First Snapshot 6



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [7,8,9,10,11] + [12,13]

Breadth-First Snapshot 7

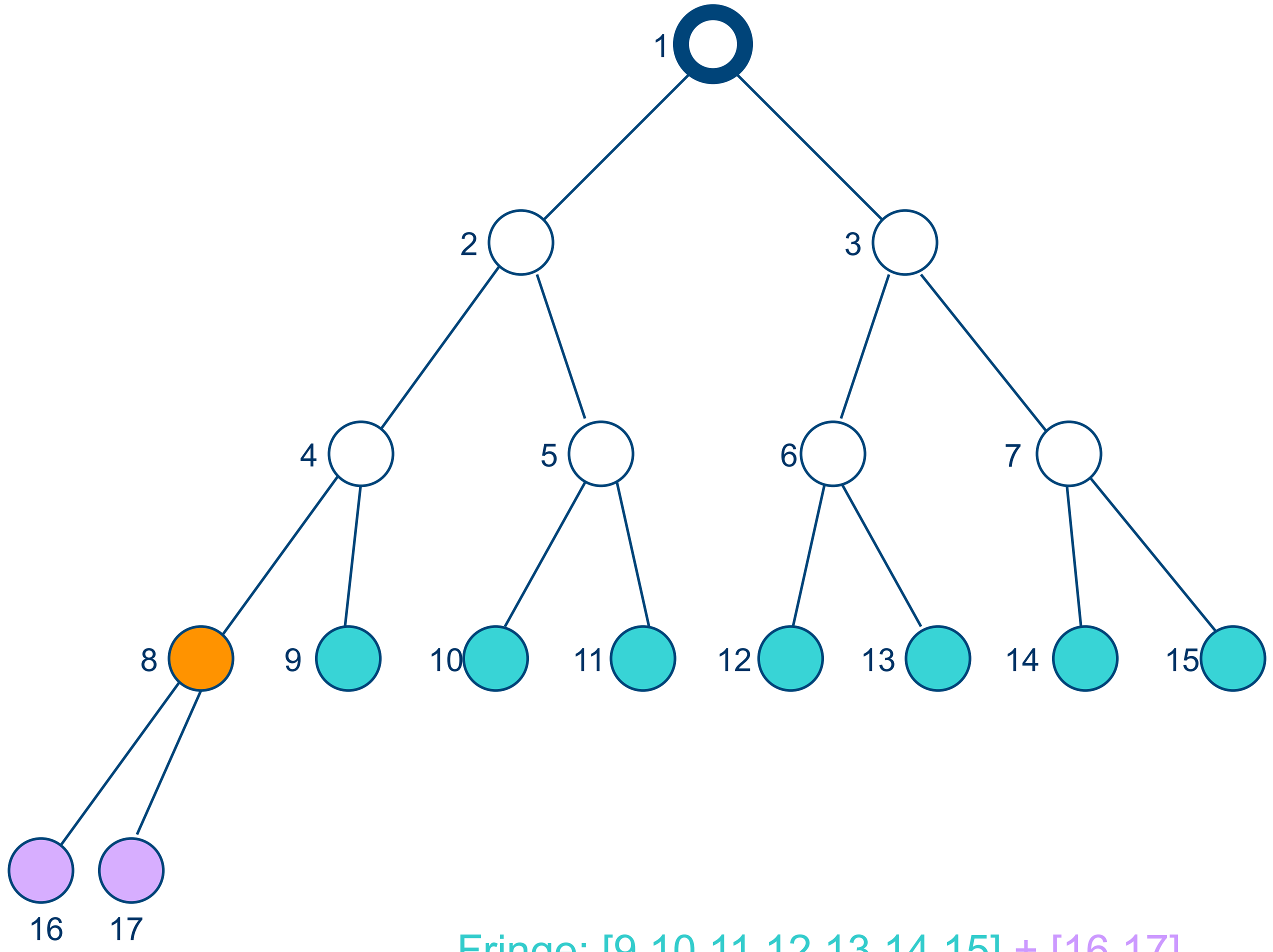


Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [8,9.10,11,12,13] + [14,15]

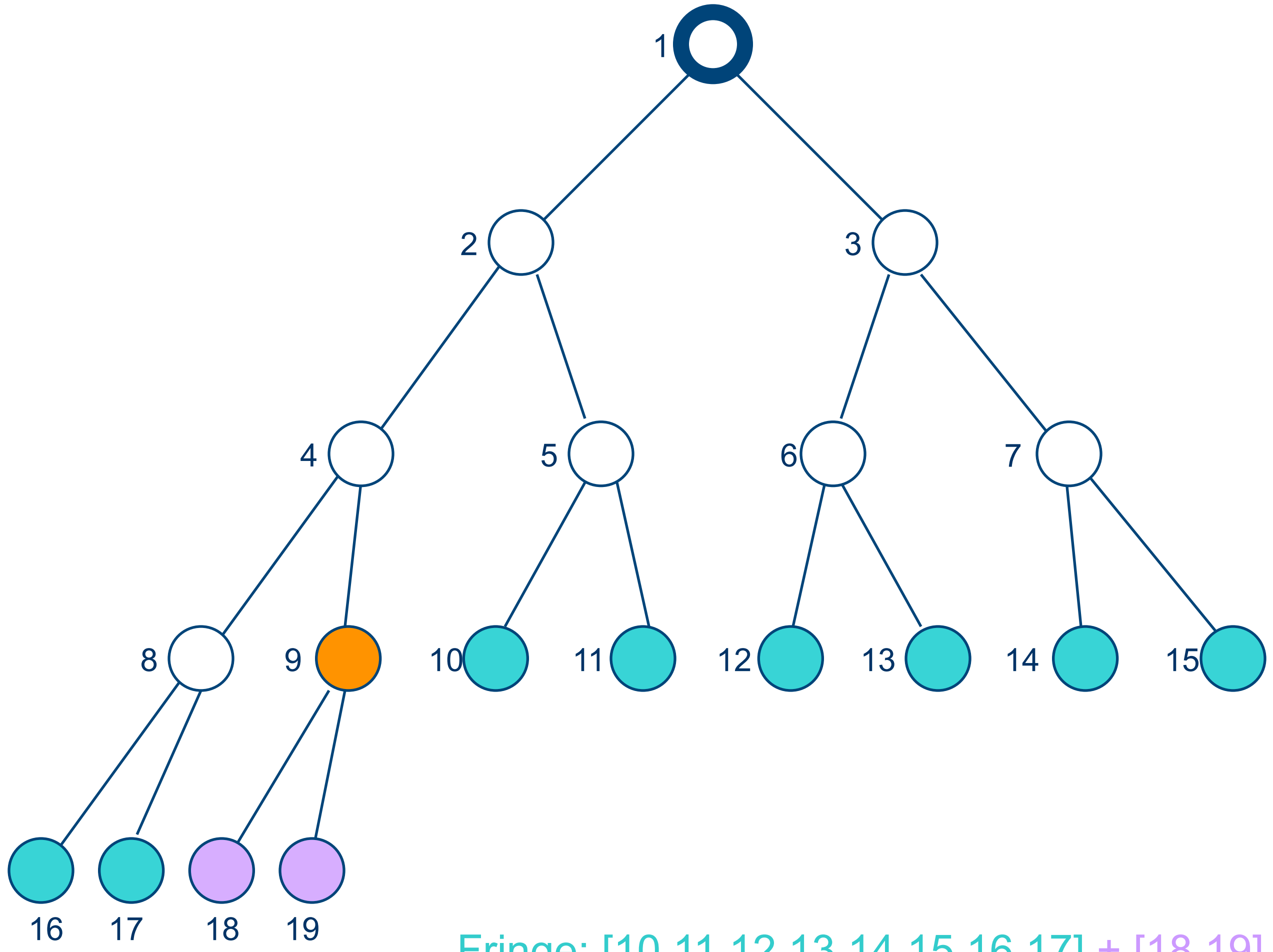
Breadth-First Snapshot 8

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●



Fringe: [9,10,11,12,13,14,15] + [16,17]

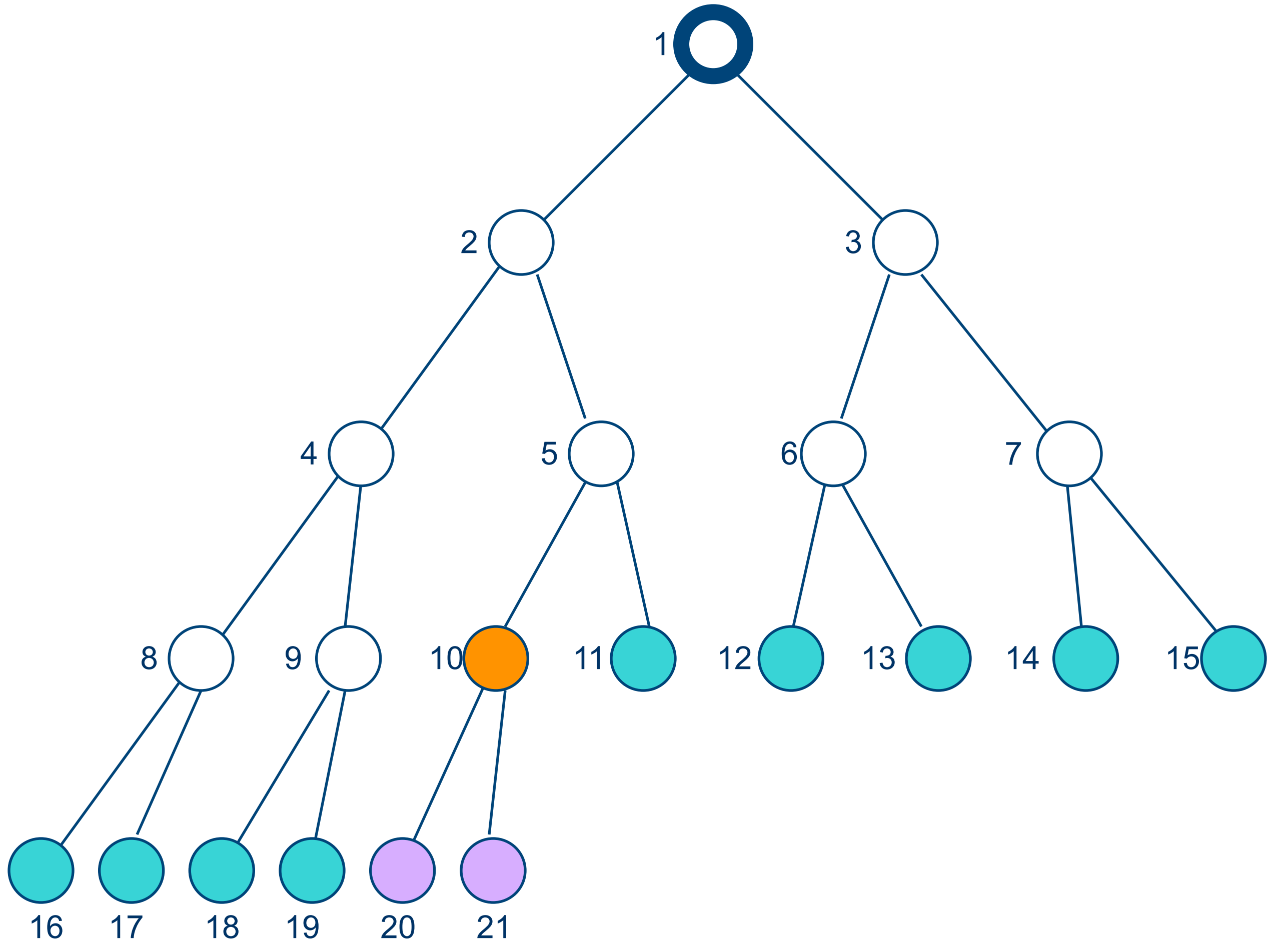
Breadth-First Snapshot 9



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [10,11,12,13,14,15,16,17] + [18,19]

Breadth-First Snapshot 10

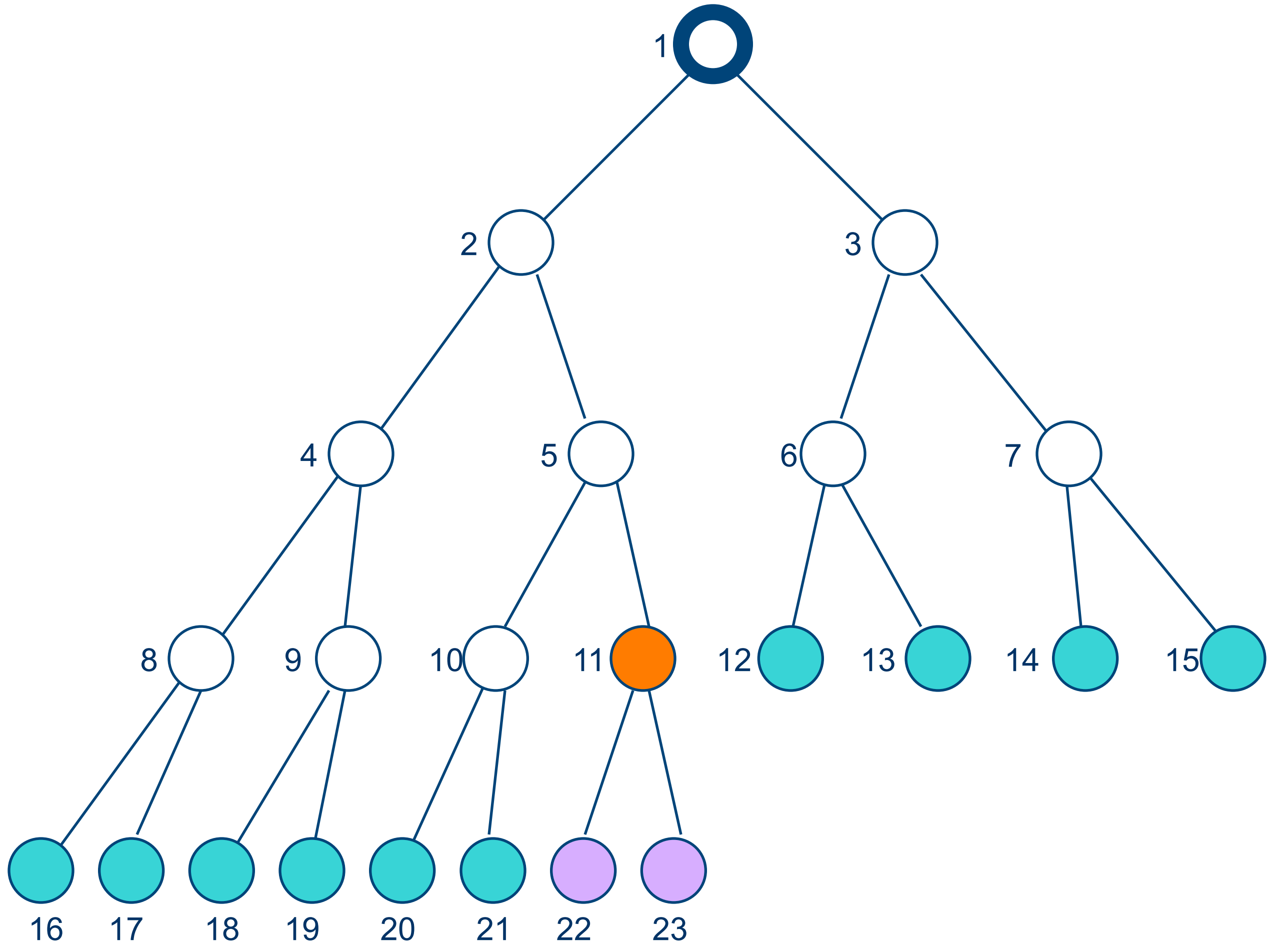


Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [11,12,13,14,15,16,17,18,19] + [20,21]

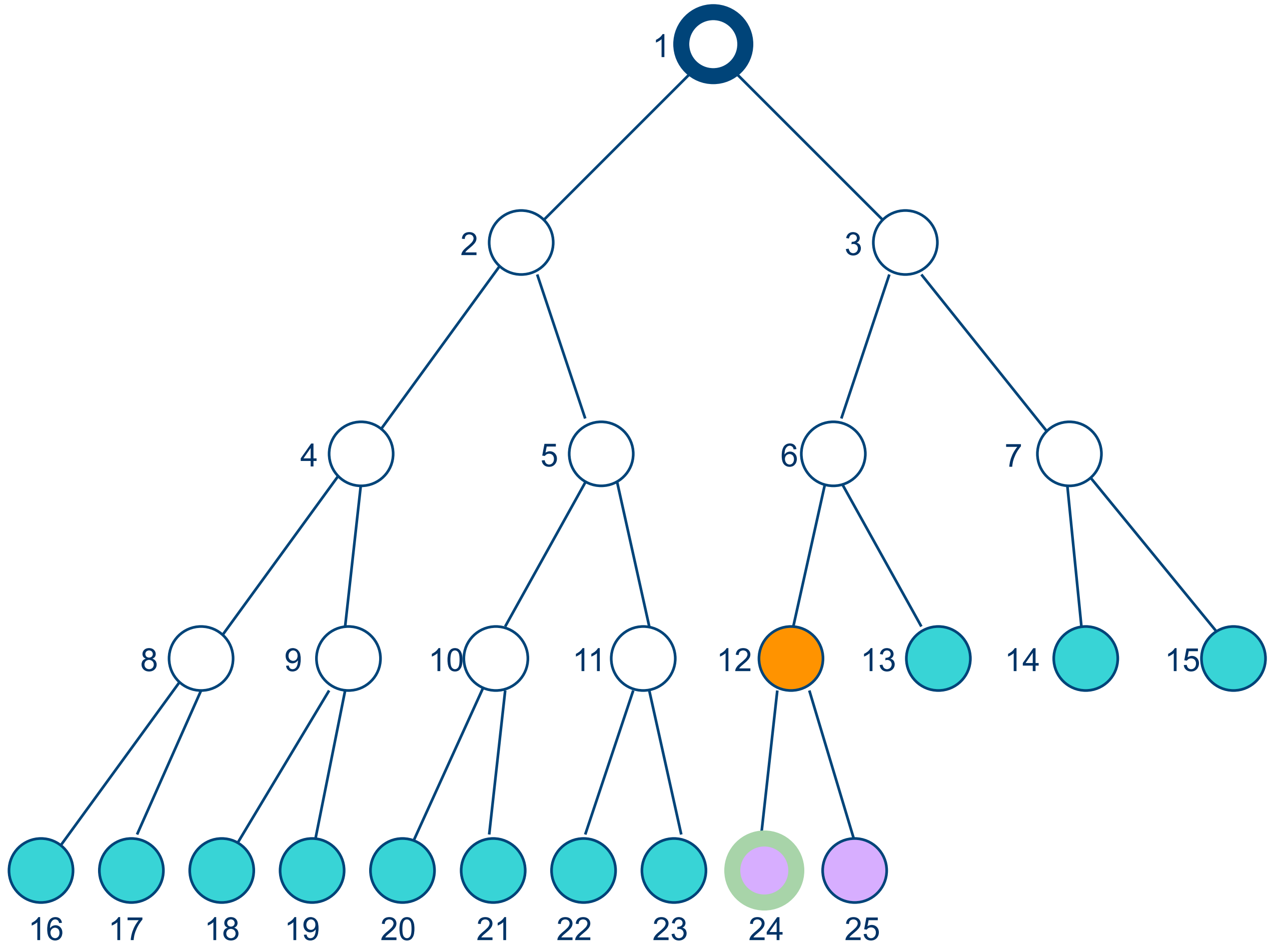
Breadth-First Snapshot 11

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●



Fringe: [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [22,23]

Breadth-First Snapshot 12

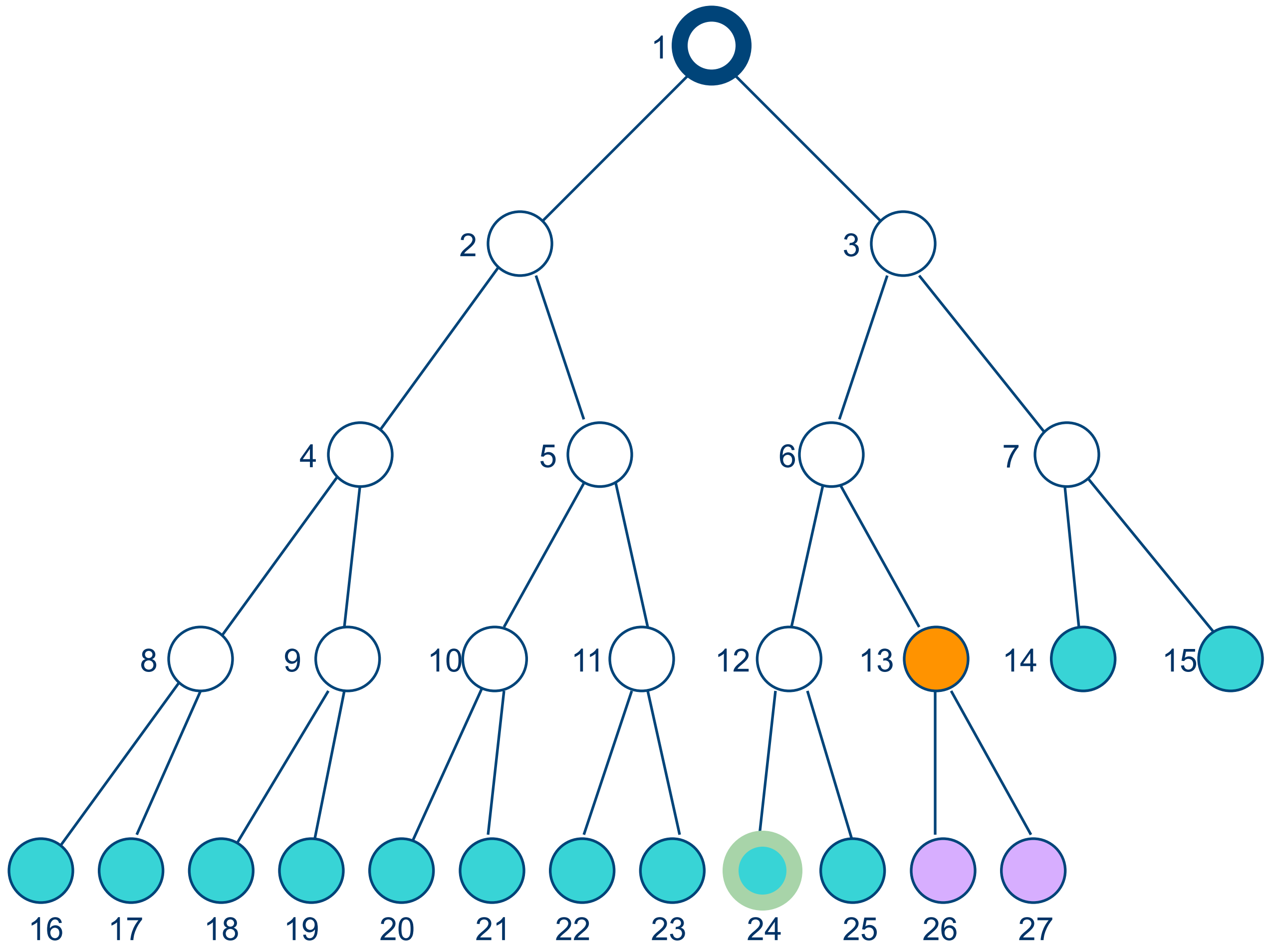


Fringe: [13,14,15,16,17,18,19,20,21] + [22,23]

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Note:
The goal node is "visible" here, but we can not perform the goal test yet.

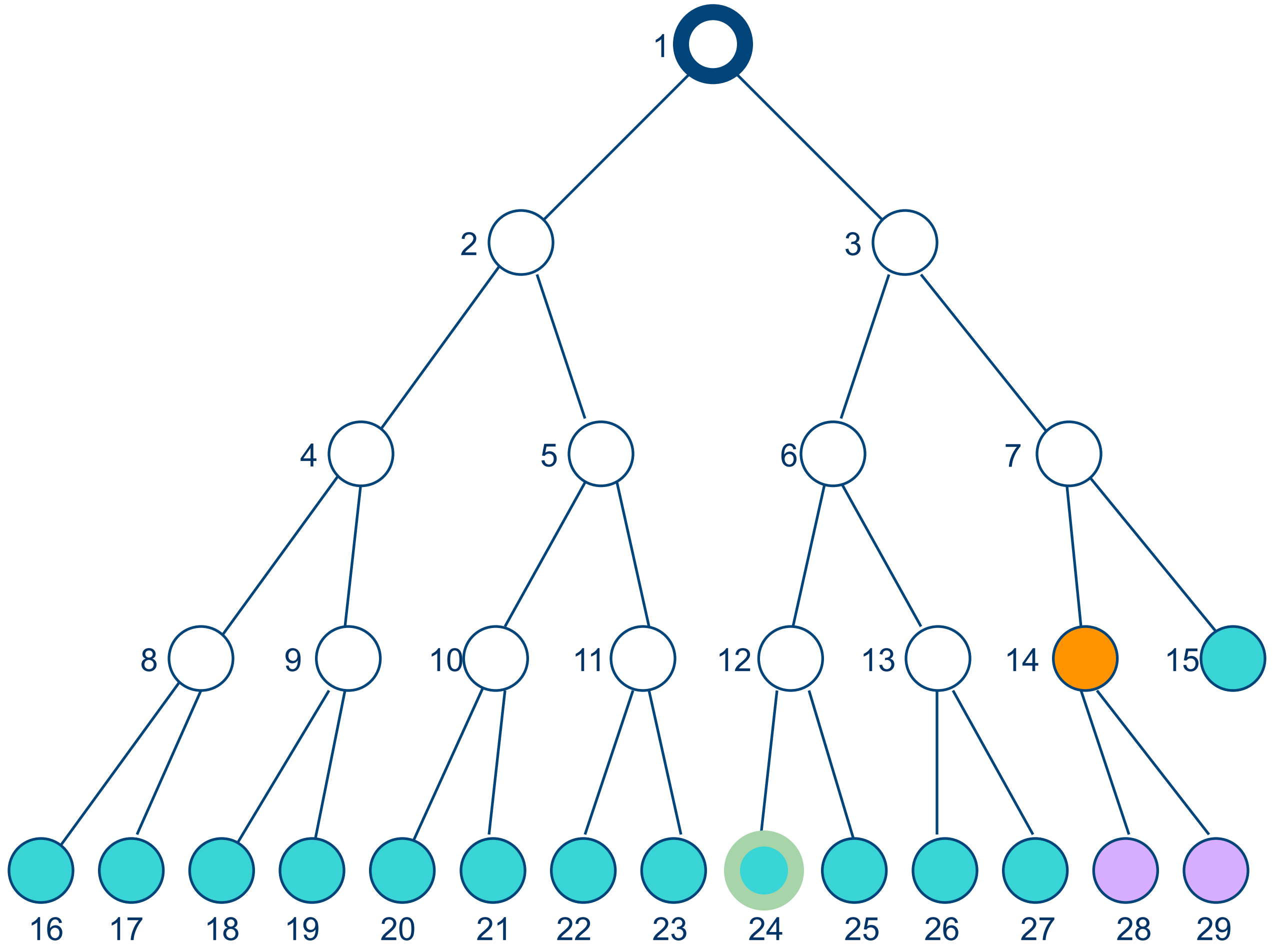
Breadth-First Snapshot 13



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [14,15,16,17,18,19,20,21,22,23,24,25] + [26,27]

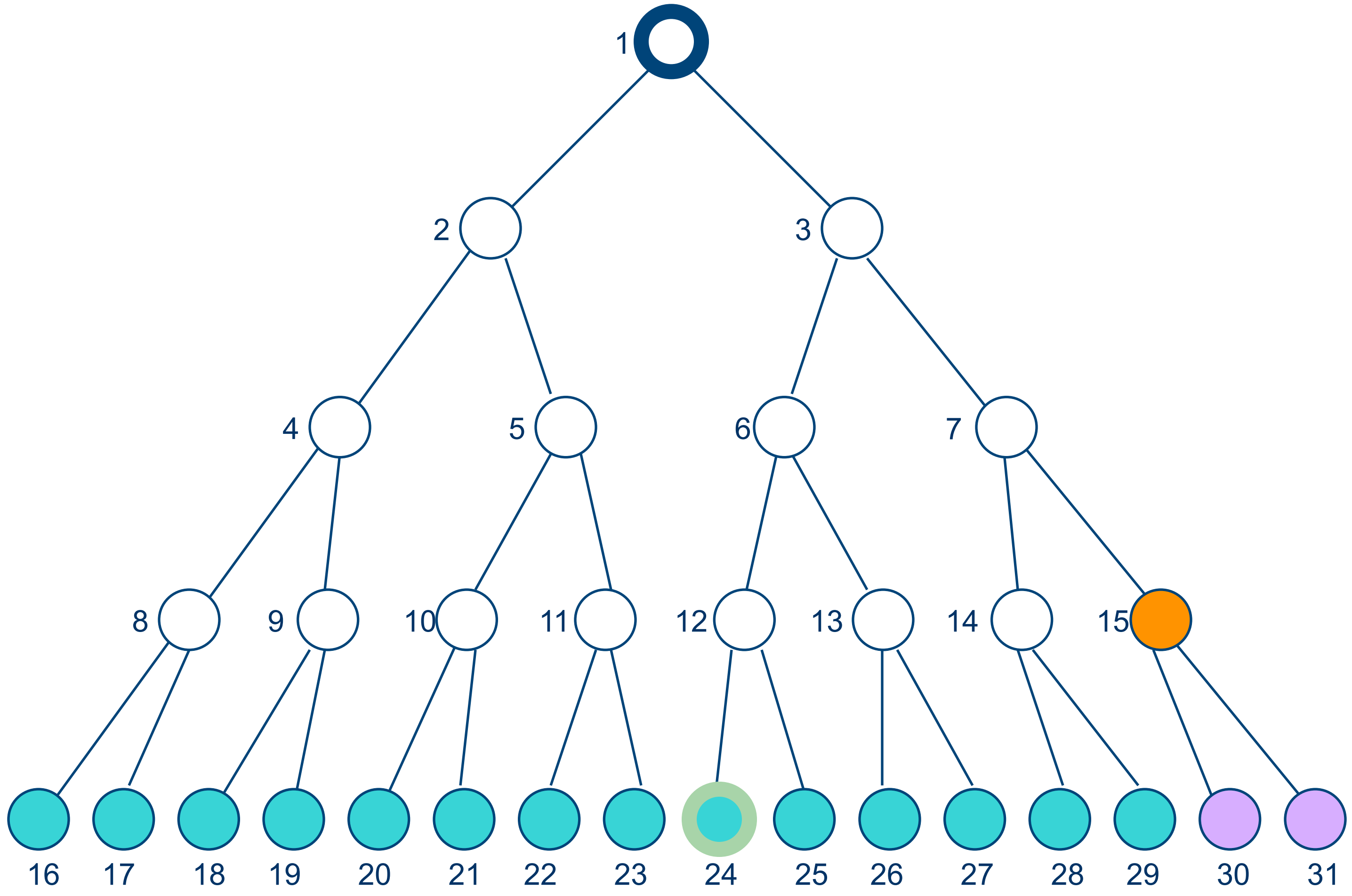
Breadth-First Snapshot 14



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [15,16,17,18,19,20,21,22,23,24,25,26,27] + [28,29]

Breadth-First Snapshot 15

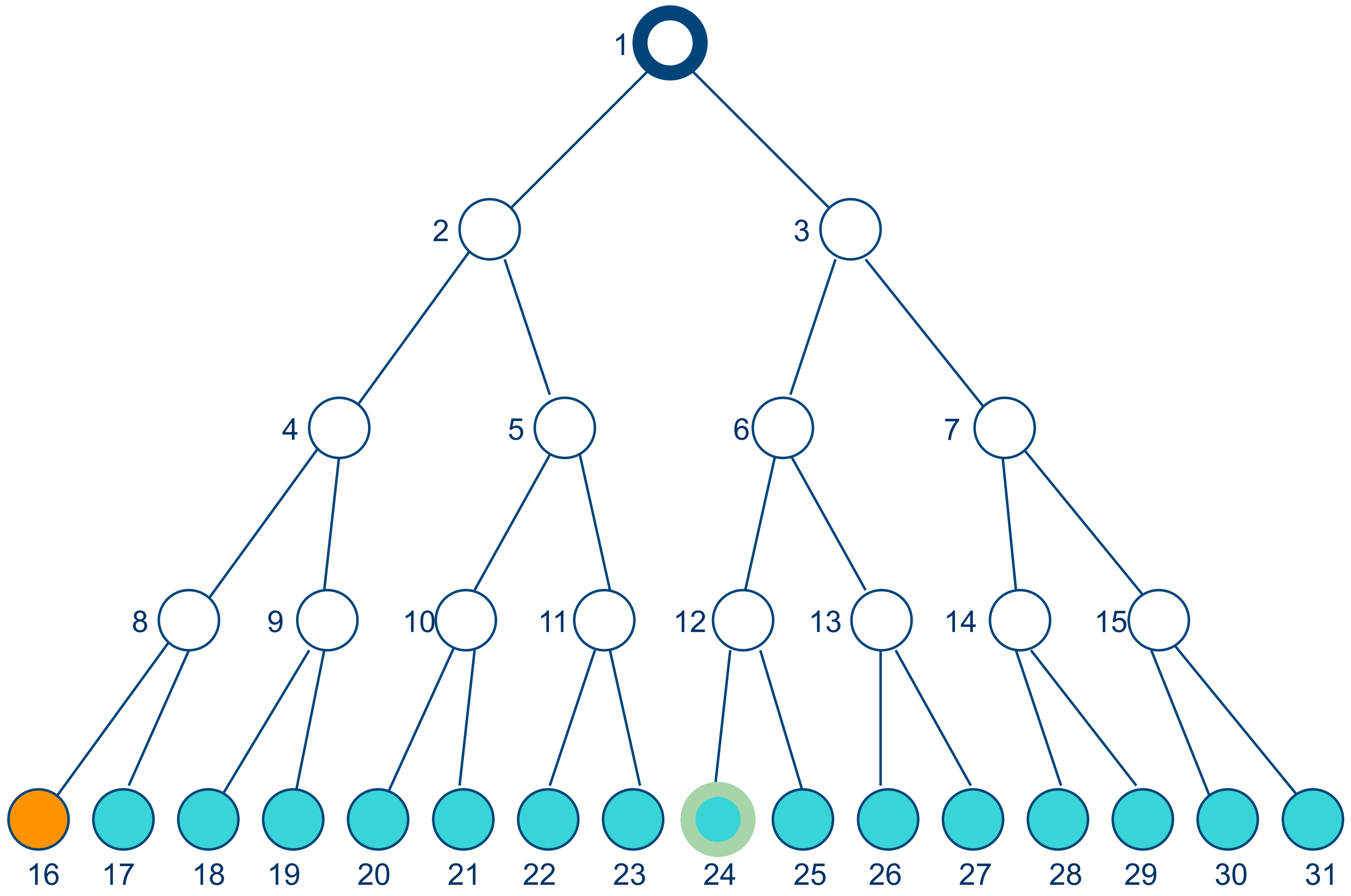


Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [15,16,17,18,19,20,21,22,23,24,25,26,27,28,29] + [30,31]

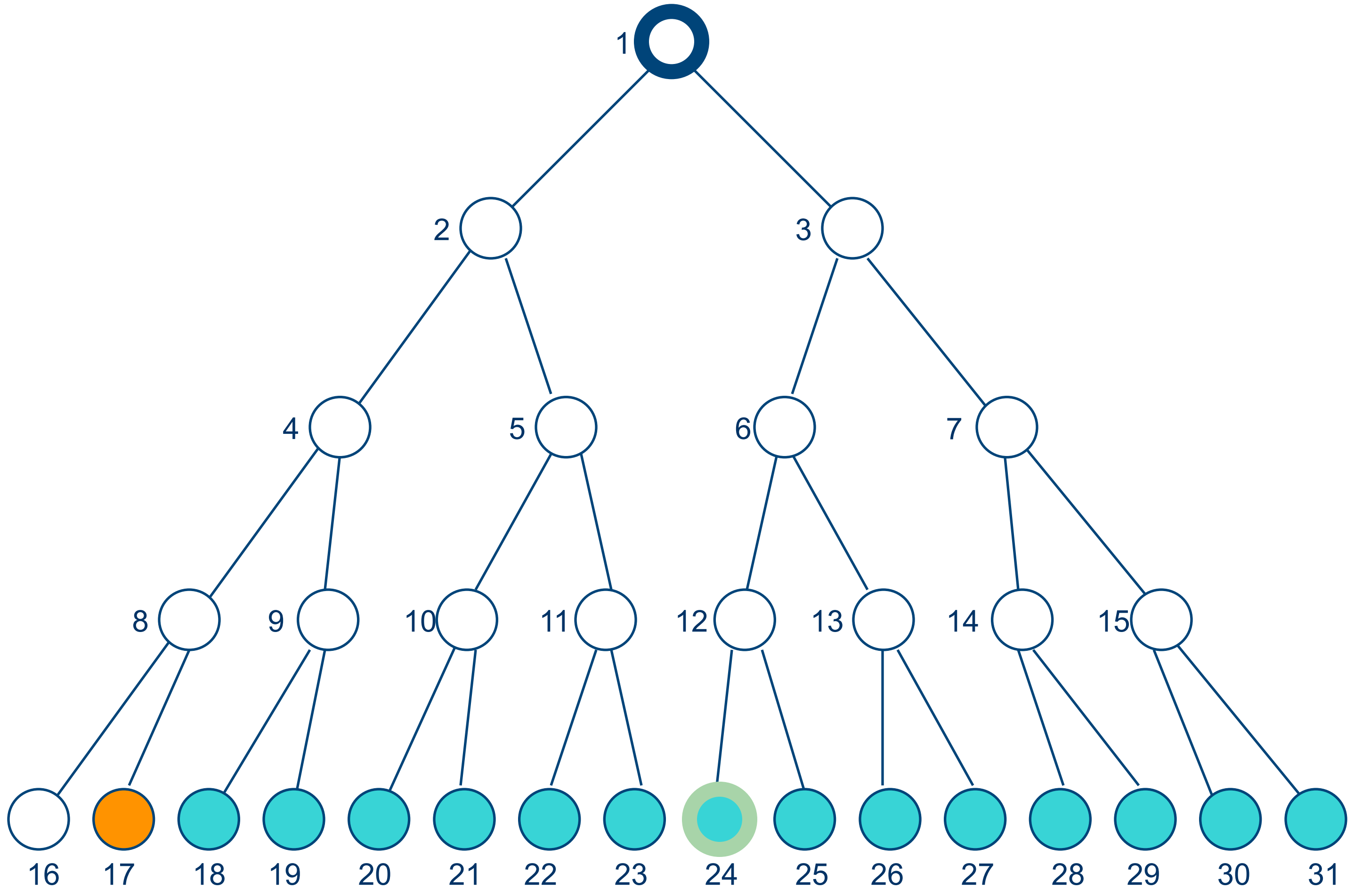
Breadth-First Snapshot 16

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●



Fringe: [17,18,19,20,21,22,23,24,25,26,27,28,29,30,31]

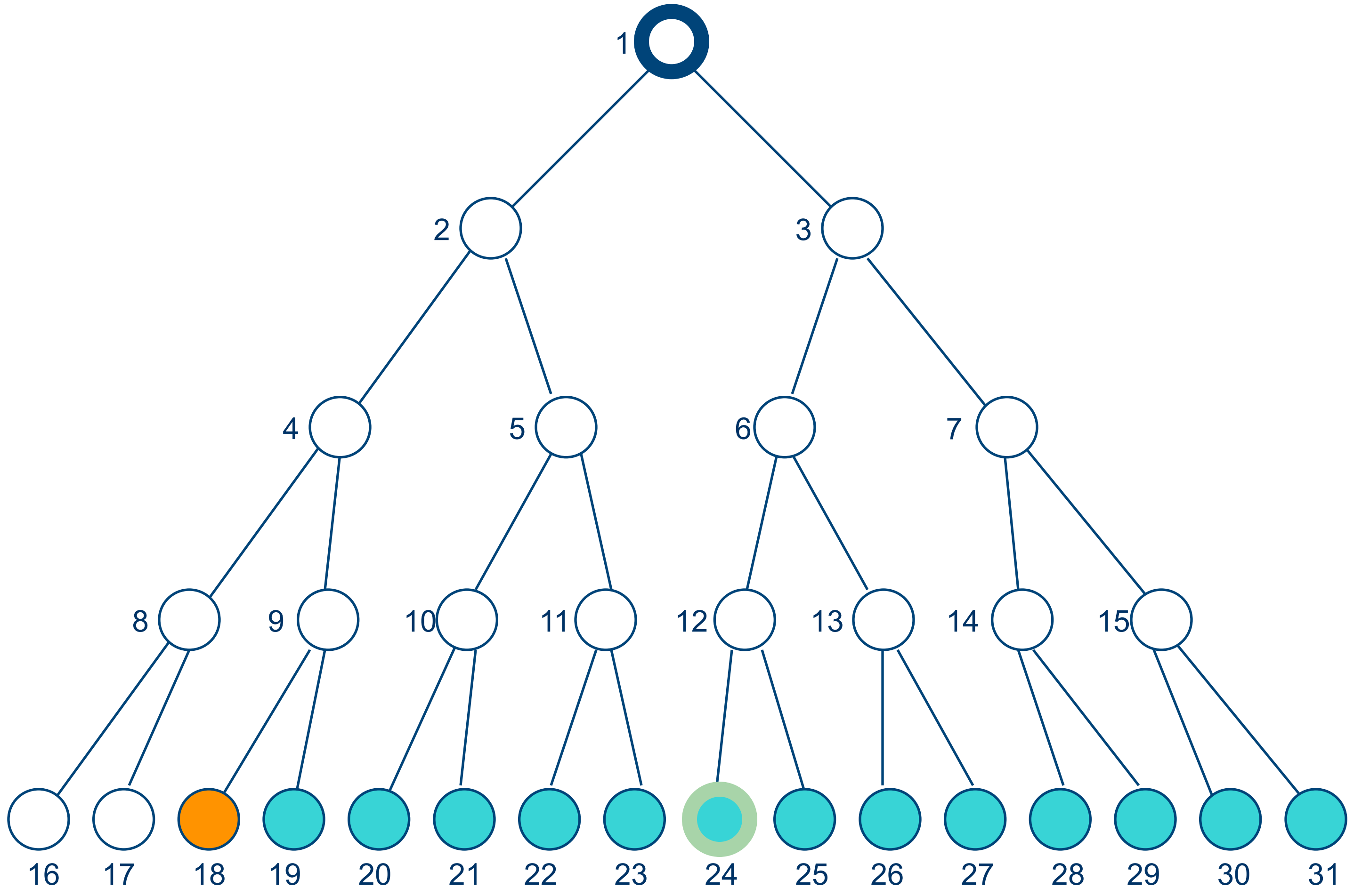
Breadth-First Snapshot 17



Fringe: [18,19,20,21,22,23,24,25,26,27,28,29,30,31]

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

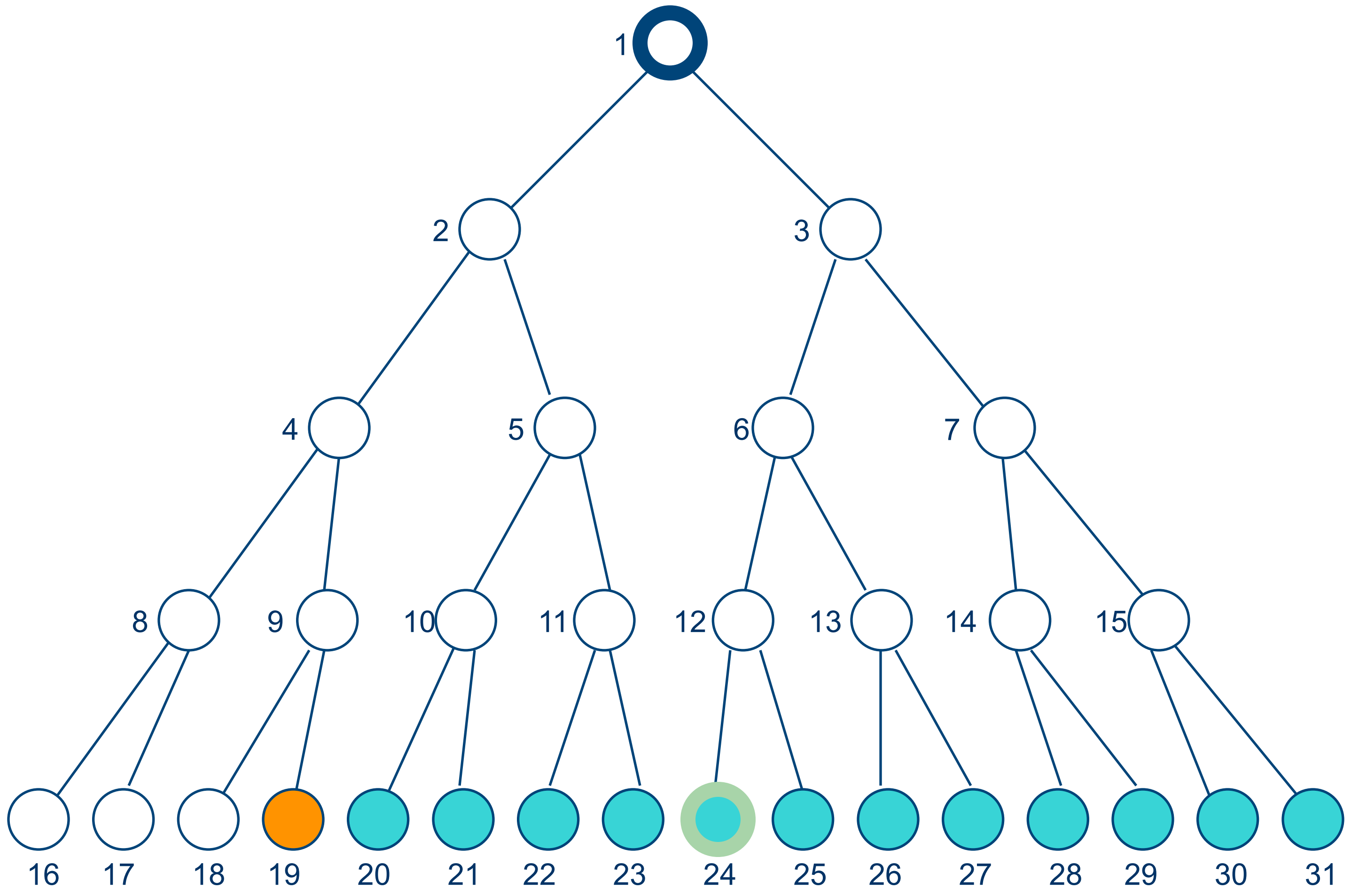
Breadth-First Snapshot 18



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [19,20,21,22,23,24,25,26,27,28,29,30,31]

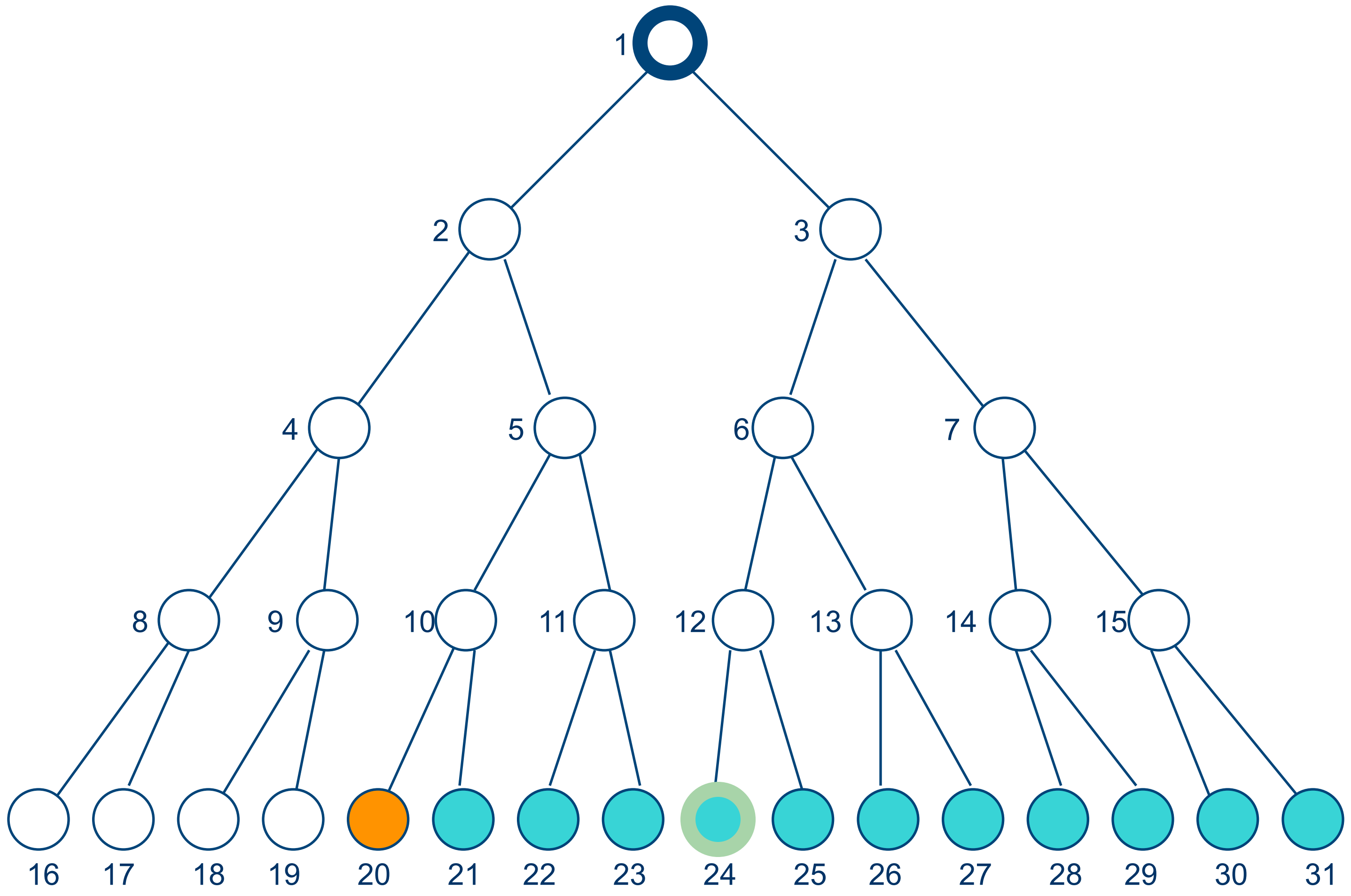
Breadth-First Snapshot 19



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [20,21,22,23,24,25,26,27,28,29,30,31]

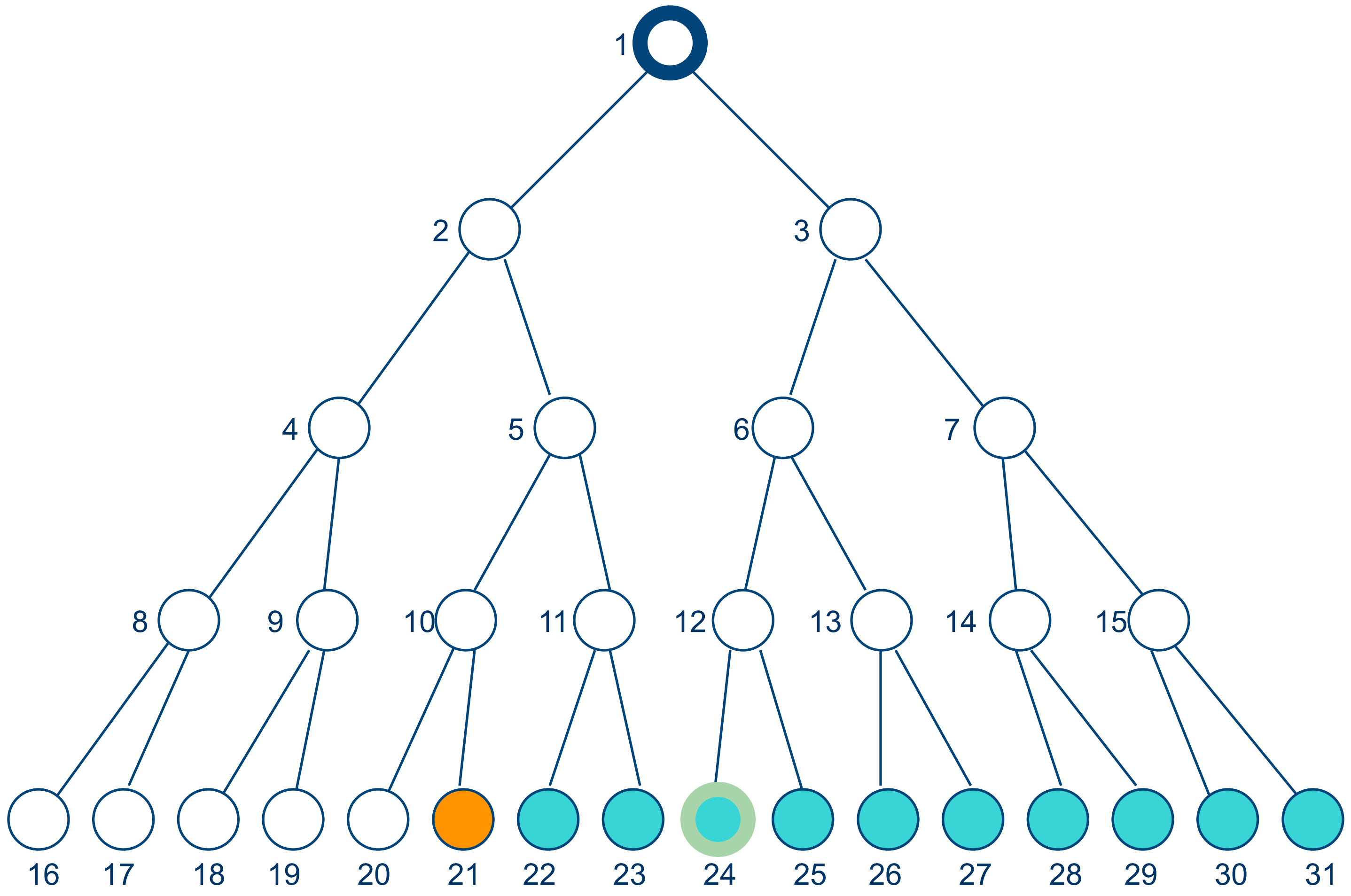
Breadth-First Snapshot 20



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [21,22,23,24,25,26,27,28,29,30,31]

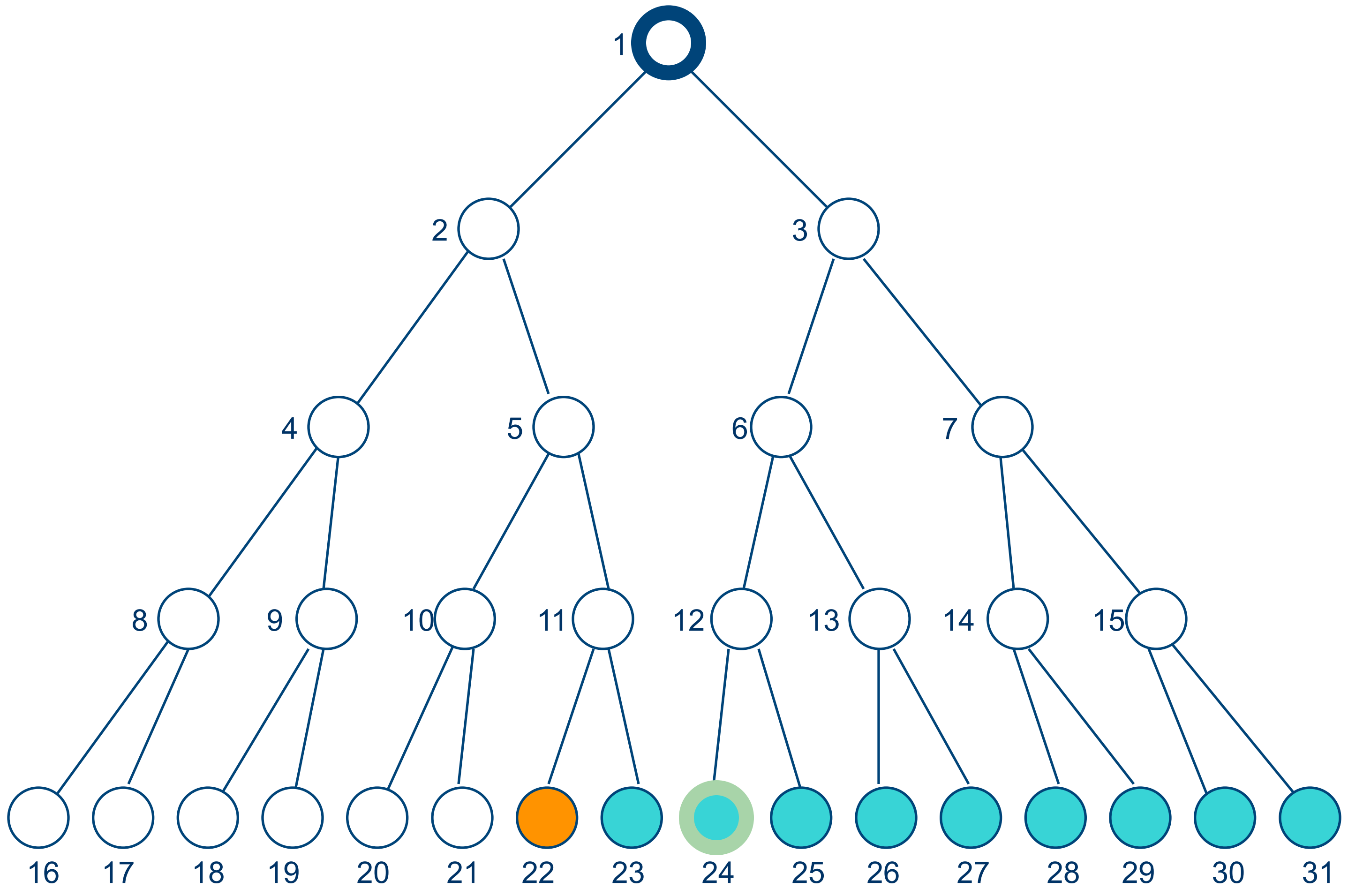
Breadth-First Snapshot 21



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [22,23,24,25,26,27,28,29,30,31]

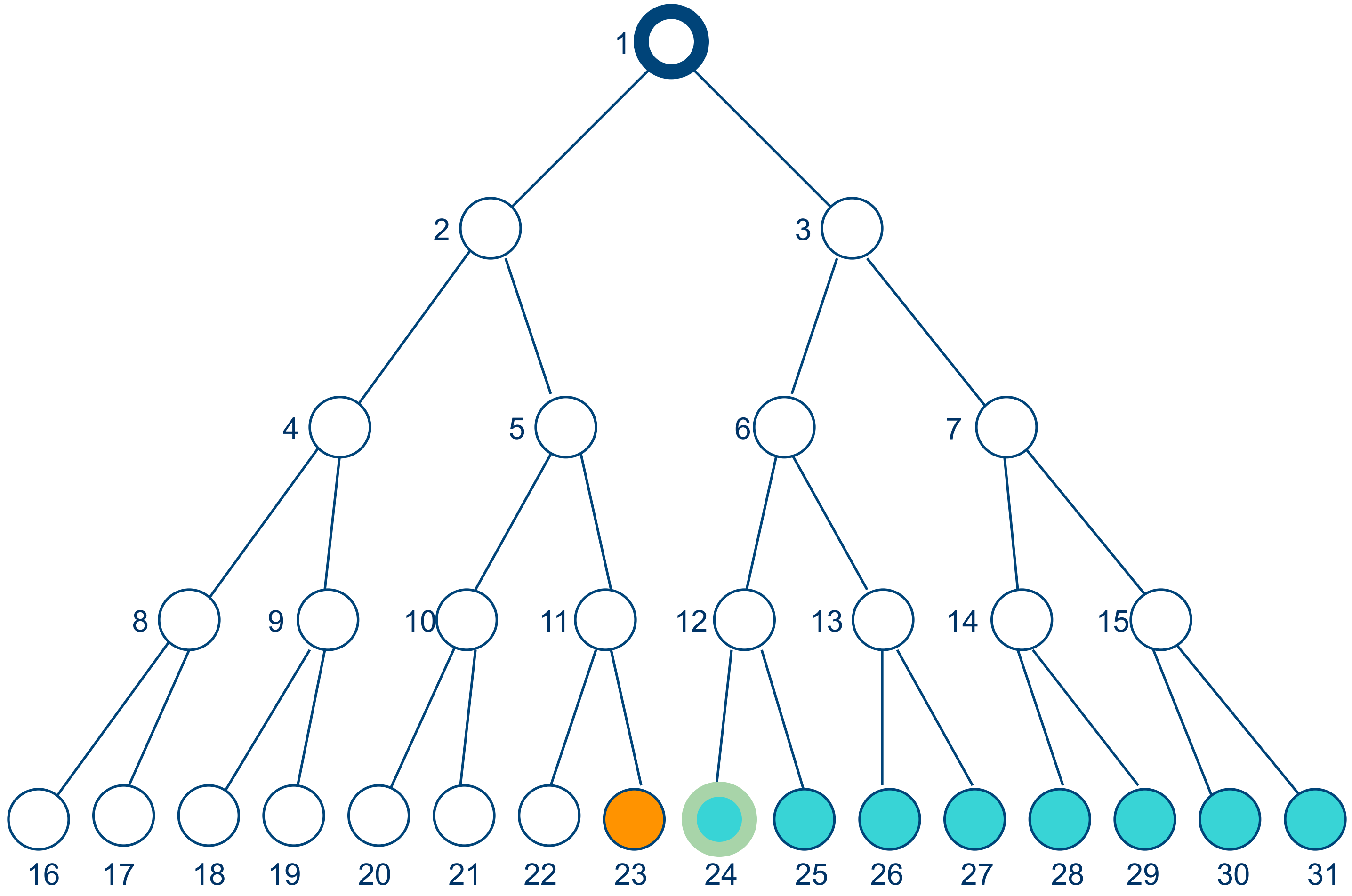
Breadth-First Snapshot 22



Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Fringe: [23,24,25,26,27,28,29,30,31]

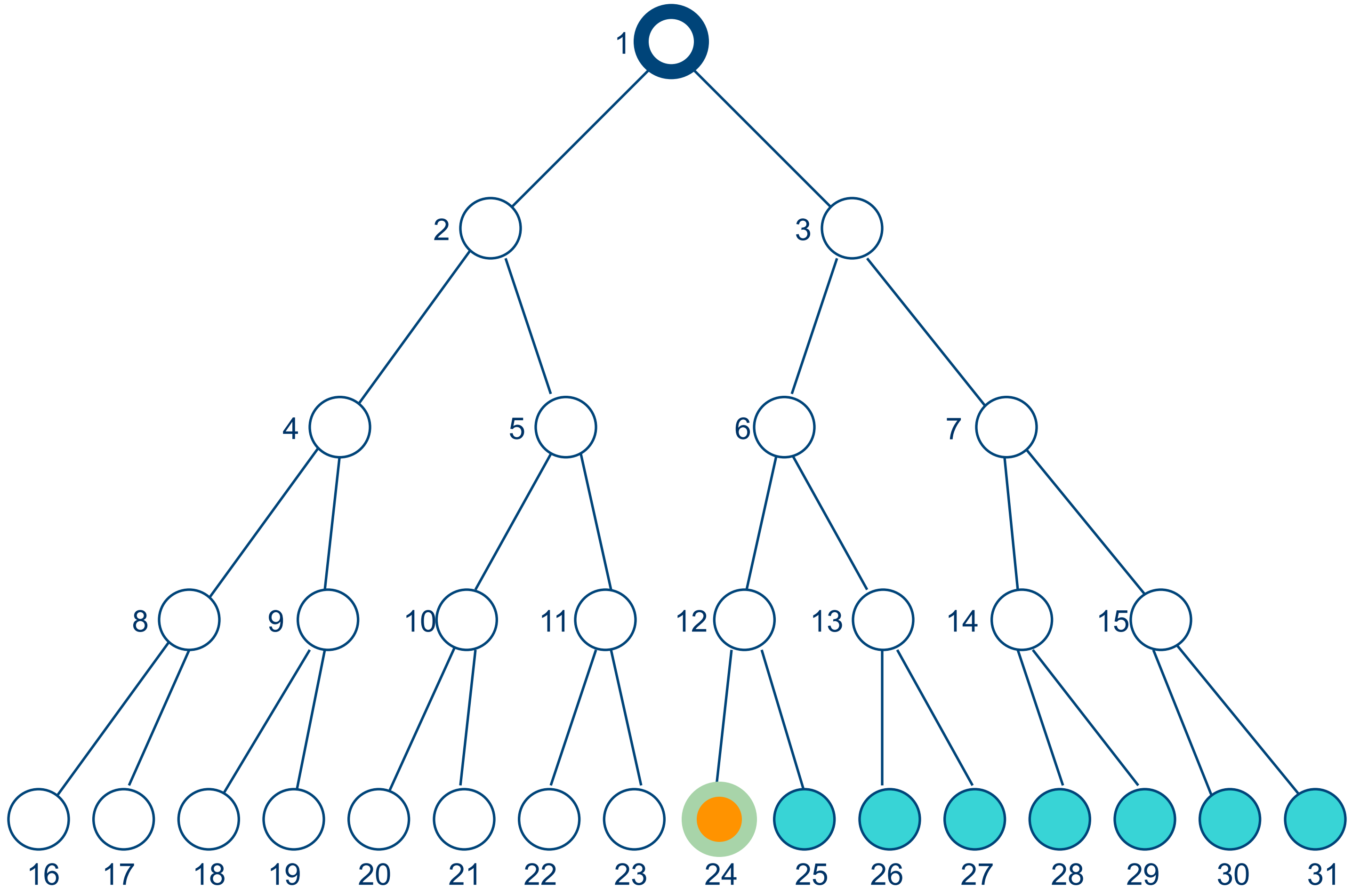
Breadth-First Snapshot 23



Fringe: [24,25,26,27,28,29,30,31]

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Breadth-First Snapshot 24



Fringe: [25,26,27,28,29,30,31]

Initial	●
Visited	○
Fringe	●
Current	●
Visible	●
Goal	●

Note:
The goal test is positive for this node, and a solution is found in 24 steps.

Breadth First Search

- Complete
- Optimal if cost is increasing with path depth.
- Computational complexity $O(b^d)$, where b is the branching factor and d is the depth
- Space (memory) complexity $O(b^d)$

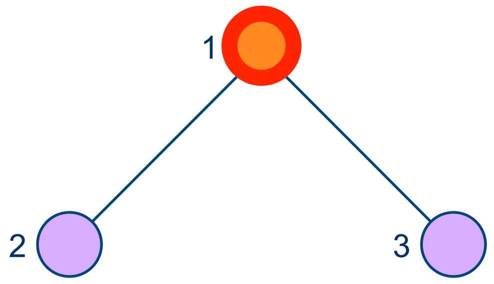
Tree Search

- Search Algorithms
  1. Breadth First Search
  2. Depth First Search
  3. A*

Depth-First

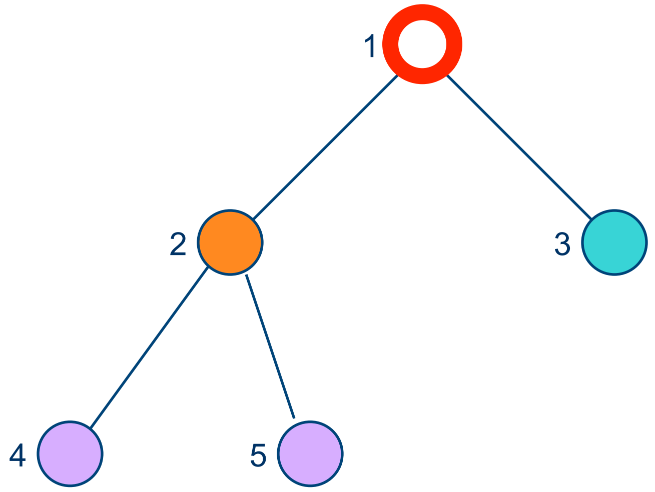
- Expands one of the nodes at the deepest level of the tree

Depth-First Snapshot 1



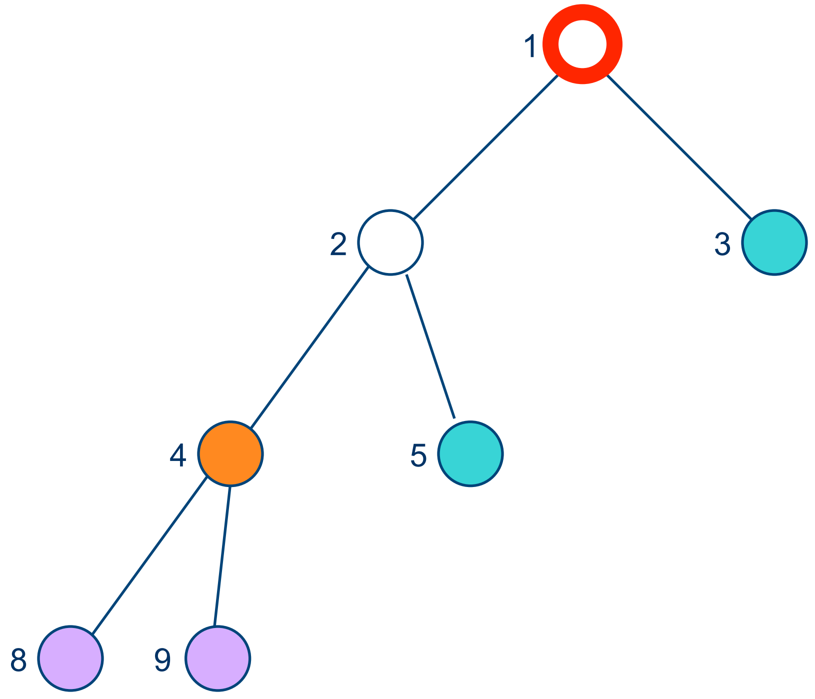
Initial	🔴
Visited	⚪
Fringe	🔵
Current	🟠
Visible	🟣
Goal	🟢

Depth-First Snapshot 2



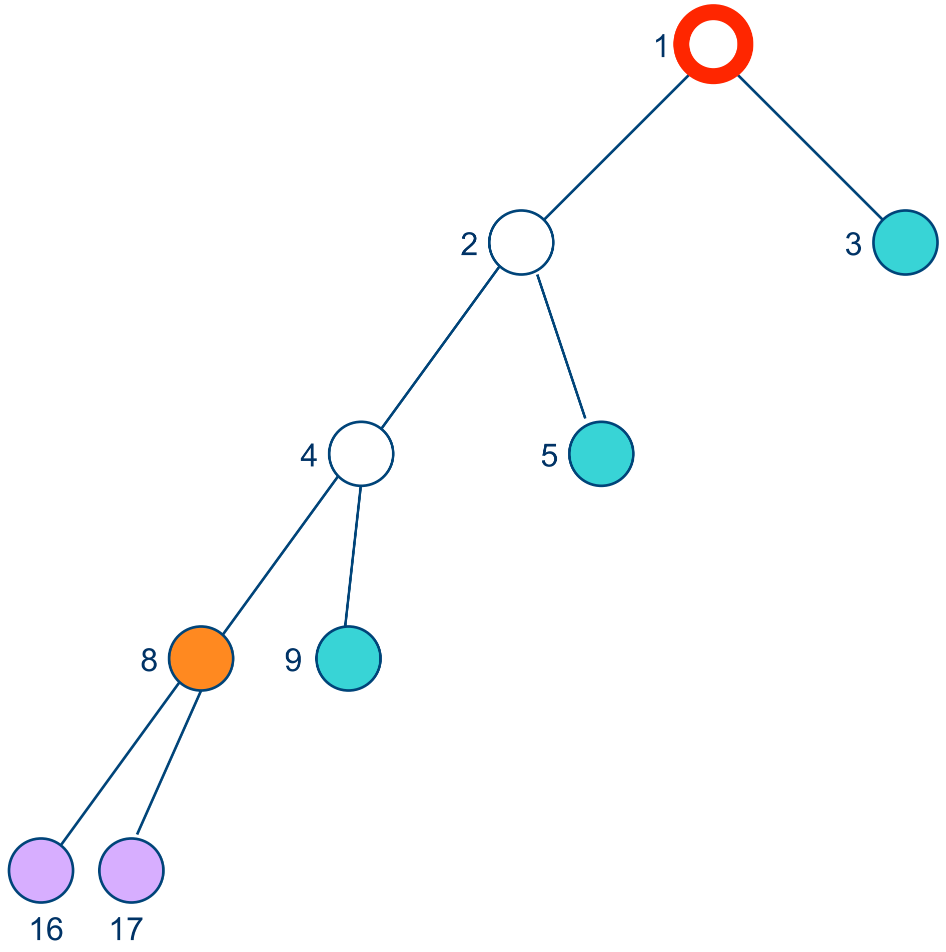
Initial	🟥
Visited	○
Fringe	●
Current	🟧
Visible	🟪
Goal	🟩

Depth-First Snapshot 3



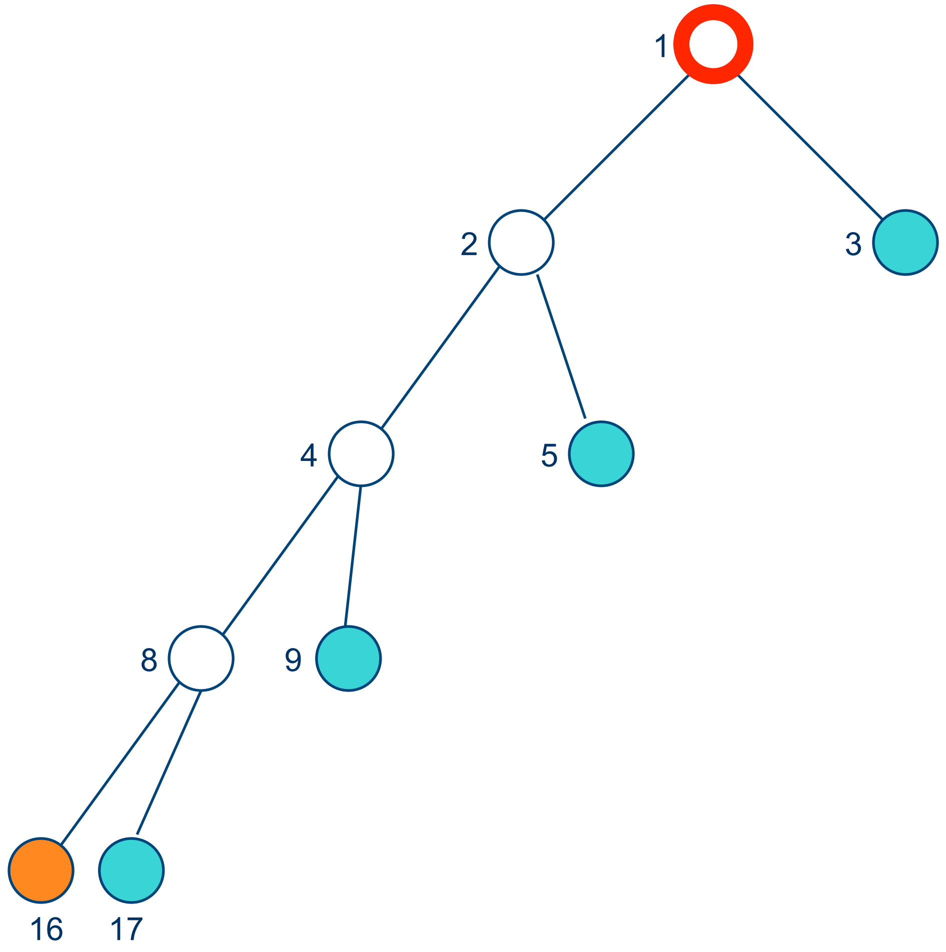
Initial	🟥
Visited	○
Fringe	●
Current	🟧
Visible	🟪
Goal	🟩

Depth-First Snapshot 4



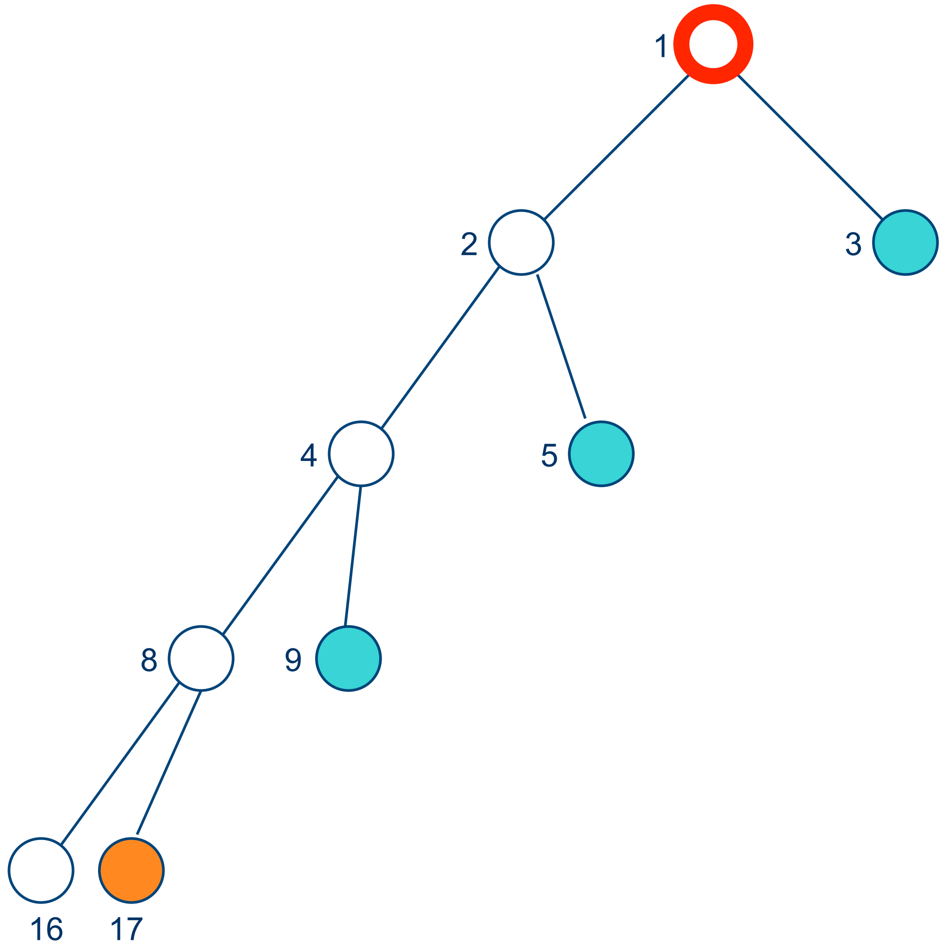
Initial	🔴
Visited	○
Fringe	●
Current	🟠
Visible	🟣
Goal	🟢

Depth-First Snapshot 5



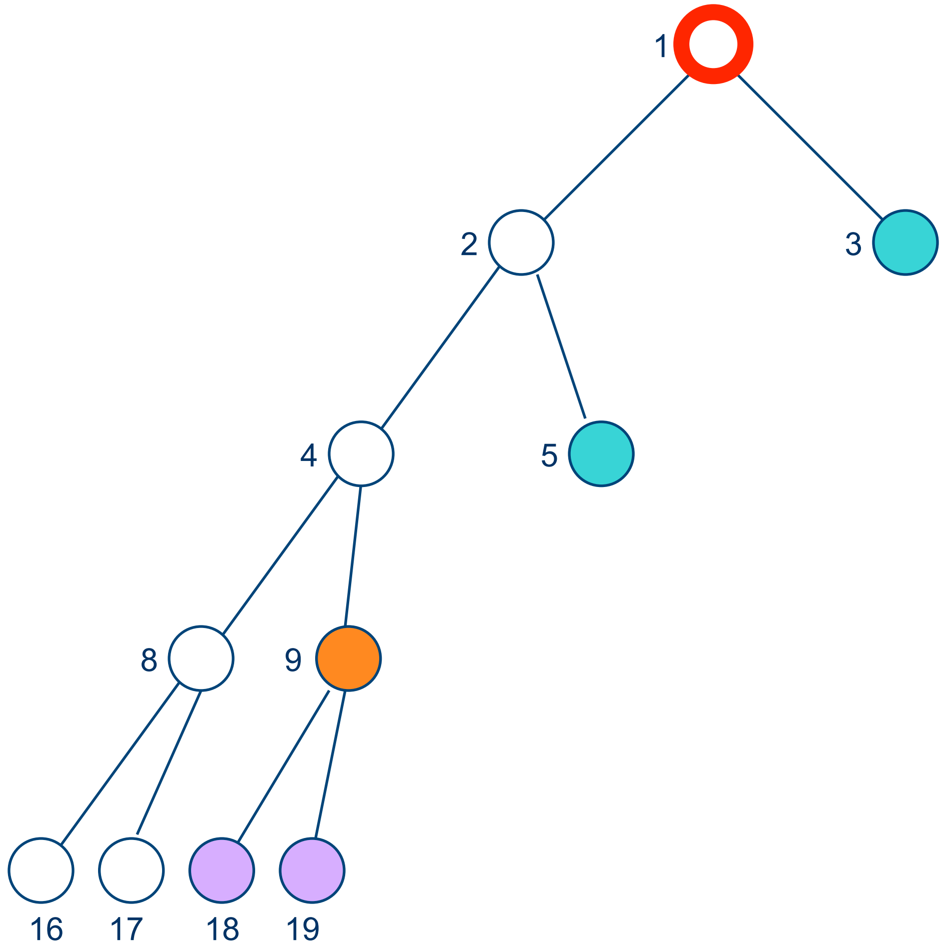
Initial	🔴
Visited	⚪
Fringe	🔵
Current	🟠
Visible	🟣
Goal	🟢

Depth-First Snapshot 6



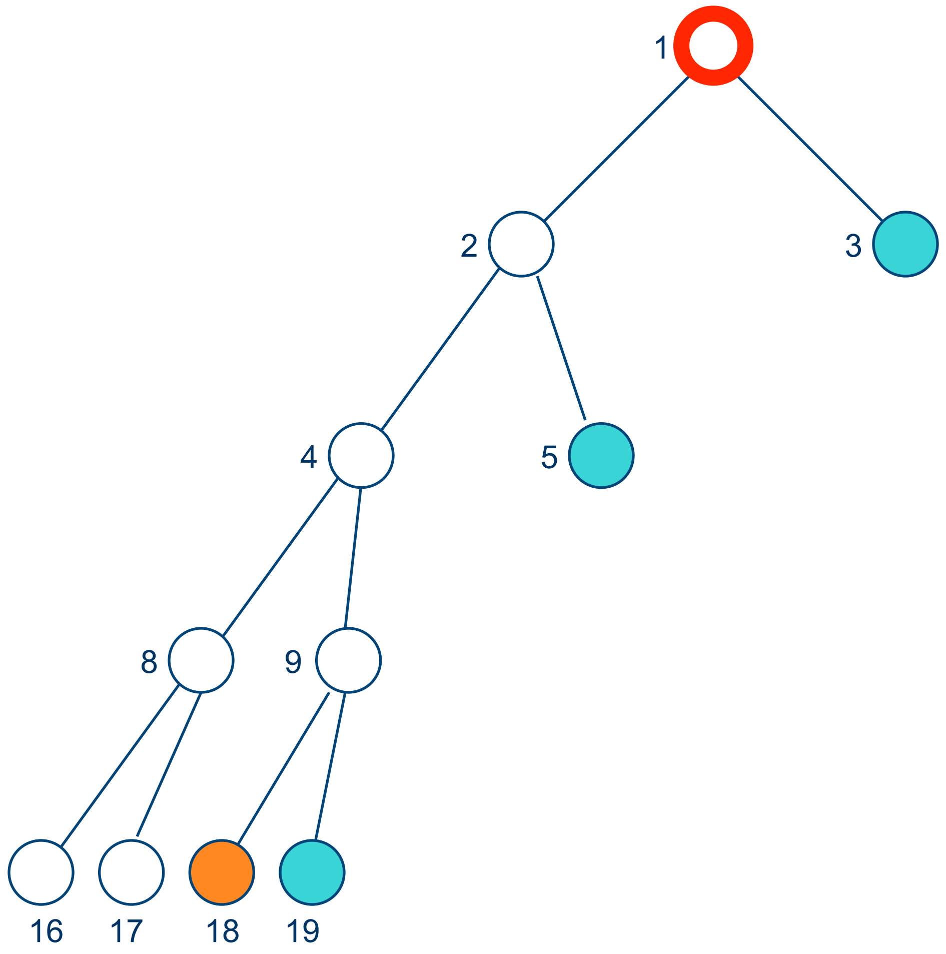
Initial	🟥
Visited	○
Fringe	●
Current	🟧
Visible	🟪
Goal	🟩

Depth-First Snapshot 7



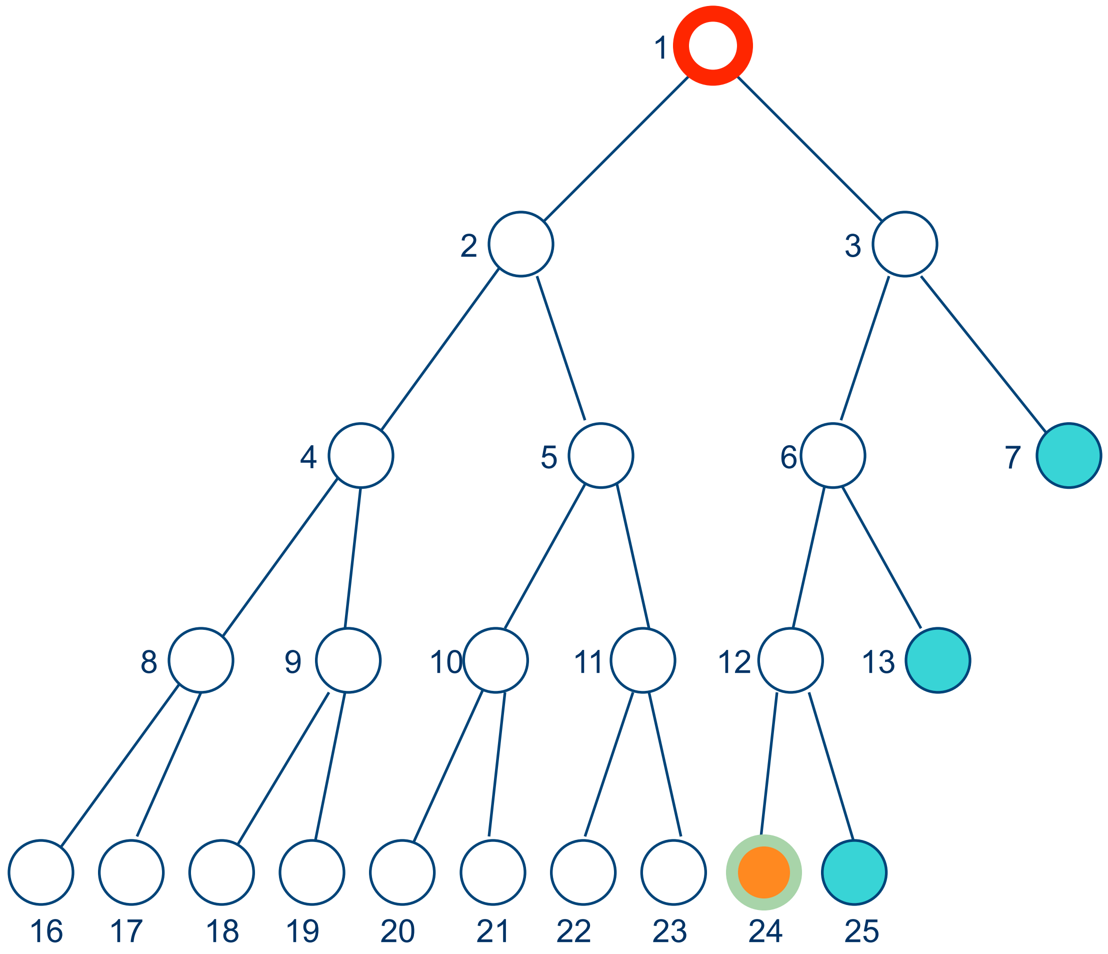
Initial	🟥
Visited	○
Fringe	●
Current	🟧
Visible	🟪
Goal	🟩

Depth-First Snapshot 8



Initial	🔴
Visited	⚪
Fringe	🔵
Current	🟠
Visible	🟣
Goal	🟢

Depth-First Snapshot …



Initial	🔴
Visited	⚪
Fringe	🔵
Current	🟠
Visible	🟣
Goal	🟢

Depth First Search

- Complete if finite depth
- NOT Optimal if we take first goal found
- Computational complexity $O(b^m)$, where b is the branching factor and m is the depth
- Space (memory) complexity $O(bm)$

Graph Search: Outline

- Search Algorithms
  1. Breadth First Search
  2. Depth First Search
  3. A*

Motion Planning:
A* Search

- There are a set of algorithms called “Best-First Search”
  - They try to search the children of the “best” node to expand.
- A* is a best first search algorithm
  - It attempts to make the best node the one that will find the optimal solution and do so in less time.

Motion Planning: A* Search

- A* is optimal and complete, but can take time…
    - Its complexity depends on the heuristic, but is exponential with the size of the graph.

Motion Planning: A* Search

- We evaluate a node n for expansion based on the function:

$$f(n) = g(n) + h(n)$$

- Where

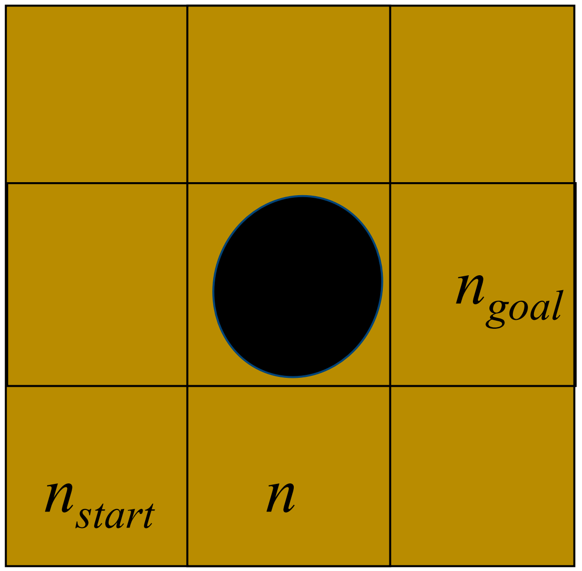
  $g(n)$ = path cost from the start node to n

  $h(n)$ = estimated cost of the cheapest path from node n to the goal

Motion Planning: A* Search

- Example: Cost for one particular node

$$f(n) = g(n) + h(n)$$

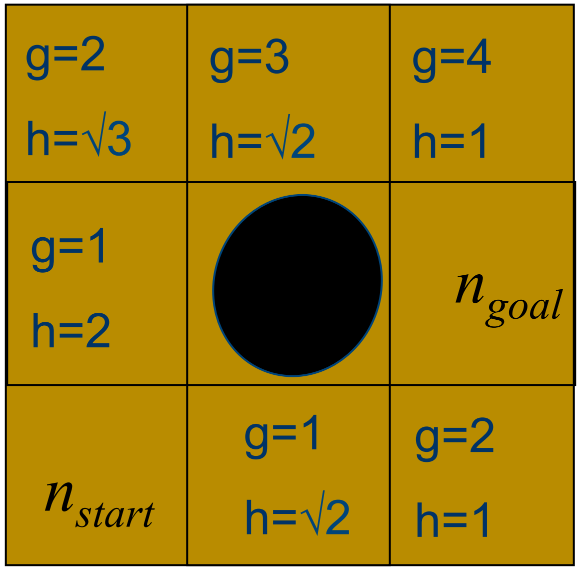


$$g(n) = 1$$
$$h(n) = \sqrt{2}$$

Motion Planning: A* Search

- Example: Cost for each node

$$f(n) = g(n) + h(n)$$



Motion Planning: A* Search

- The strategy is to expand the node with the cheapest path (lowest f).
- This is proven to be complete and optimal, if $h(n)$ is an *admissible* heuristic.

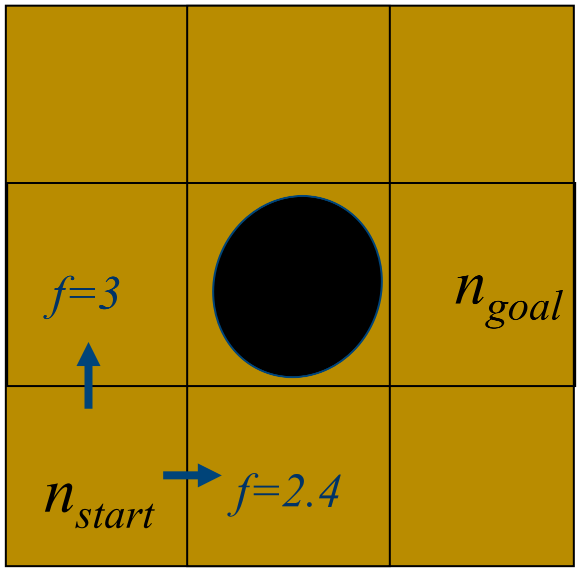
Motion Planning: A* Search

- Here, *h(n)* is an admissible heuristic is one that never *overestimates* the cost to the goal
  - Example: the Euclidean distance.

Motion Planning: A* Search

- Search example: Iteration 1

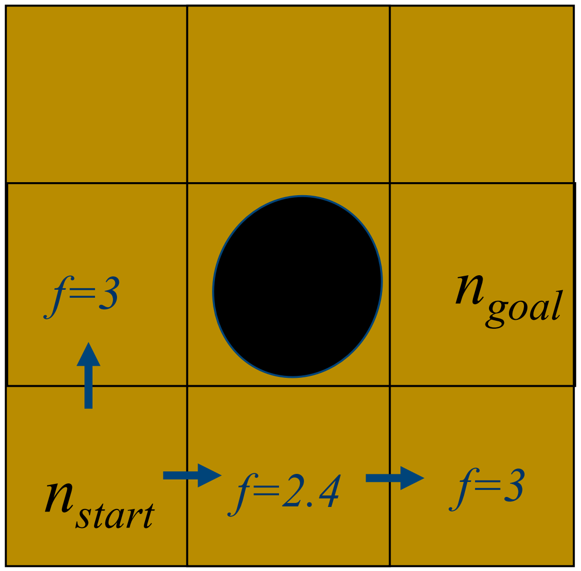
$$Fringe\ set = \{f_1 = 2.4, f_2 = 3\}$$



Motion Planning: A* Search

- Search example: Iteration 2

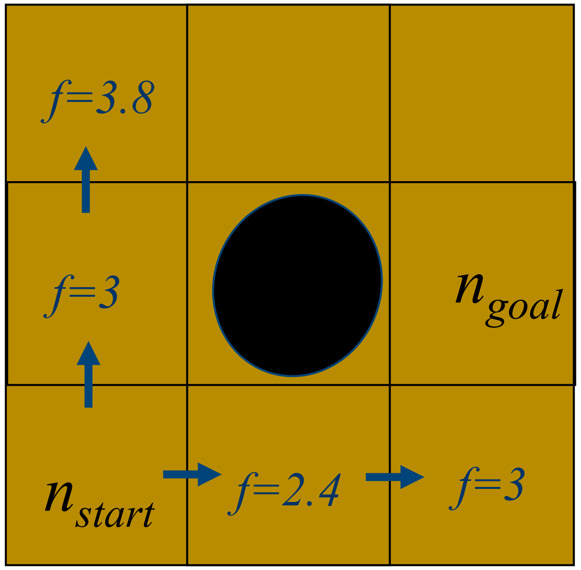
$$Fringe\ set = \{f_2 = 3, f_3 = 3\}$$



Motion Planning: A* Search

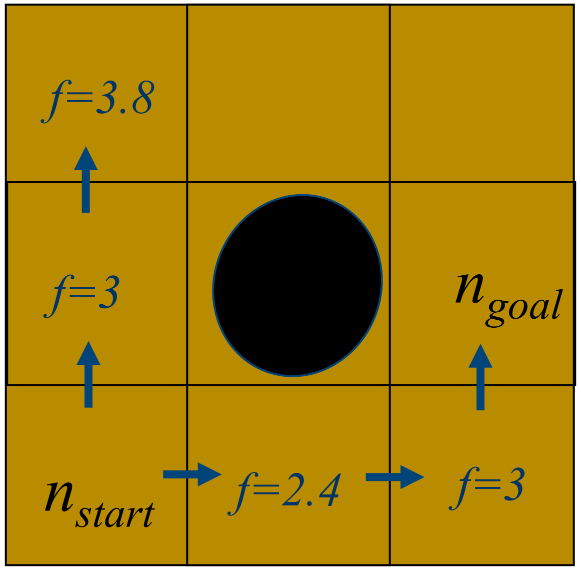
- Search example: Iteration 3

$$\text{Fringe set} = \{f_3 = 3, f_4 = 3.8\}$$



Motion Planning: A* Search

- Search example: Iteration 4



Motion Planning: Final Notes

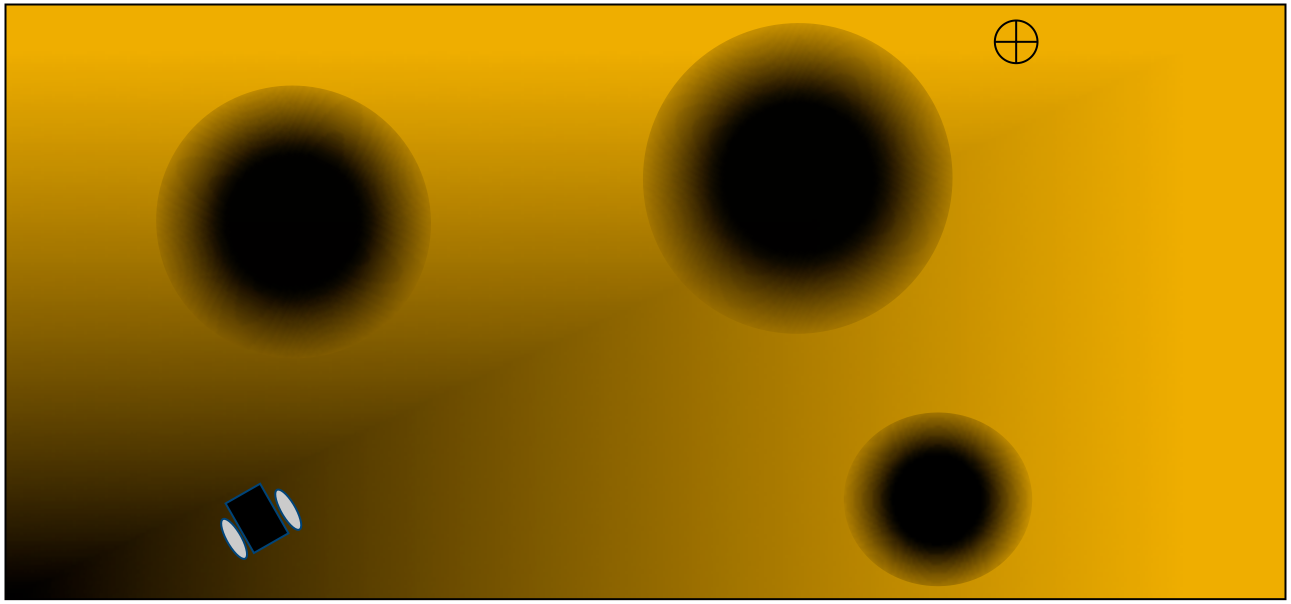
- A * is often used as a global planner
- Planner that considers kinematic/dynamic constraints is used for local planning.

MP: Outline

1. Multi-Query PRMs
2. Graph Search
3. Artificial Potential Fields

Artificial Potential Fields

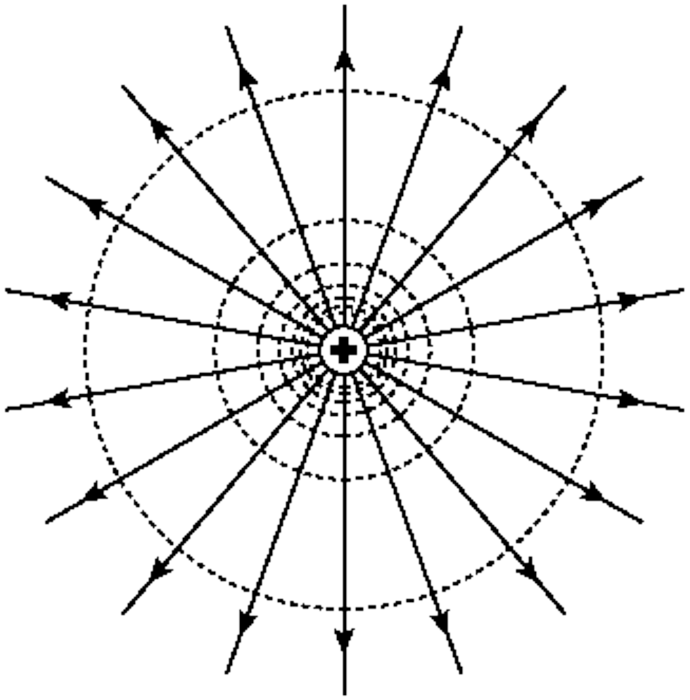
- Potential field
  - Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent



Artificial Potential Fields

- Electric Potentials
  - The electric potential V_E (J C^{-1}) created by a point charge Q, at a distance r from the charge (relative to the potential at infinity), can be shown to be

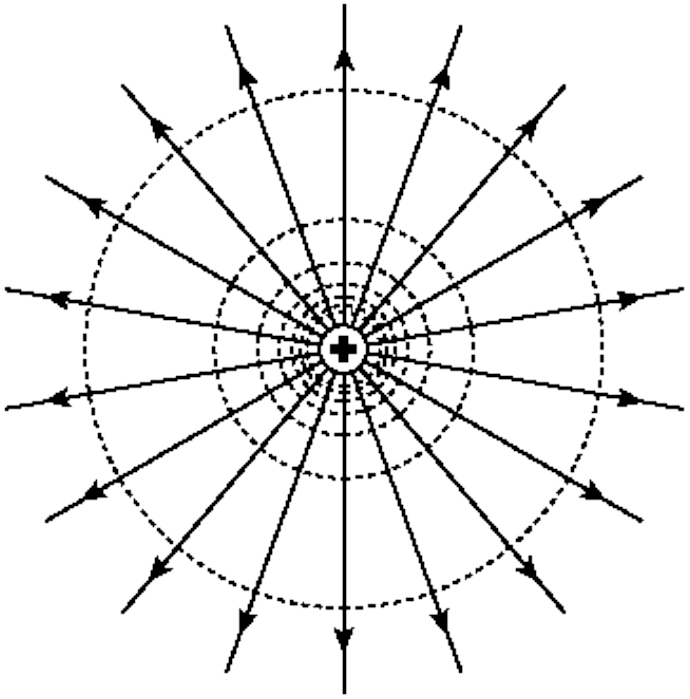
$$V_E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$



Artificial Potential Fields

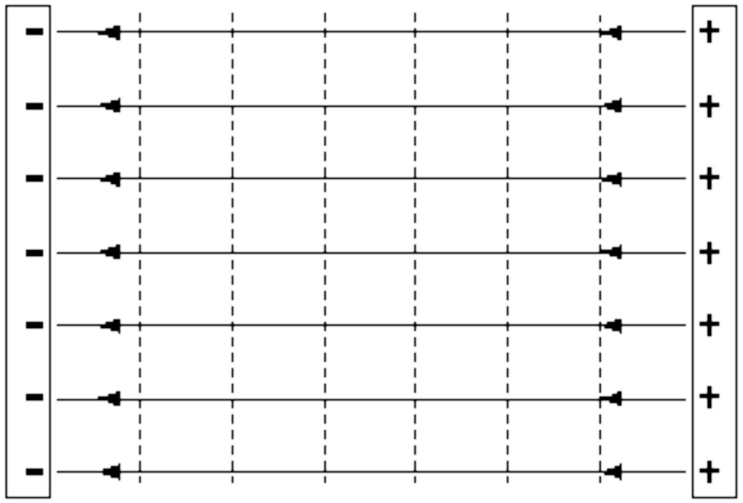
- Electric Fields
  - The electric field intensity E is defined as the force per unit positive charge that would be experienced by a point charge
  - It is obtained by taking the negative gradient of the electric potential

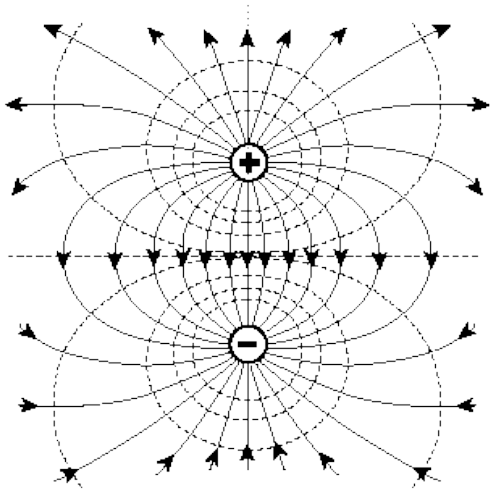
$$E = -\nabla V_E$$



Artificial Potential Fields

- Electric Potential Fields
  - Different arrangements of charges can lead to various fields





Artificial Potential Fields

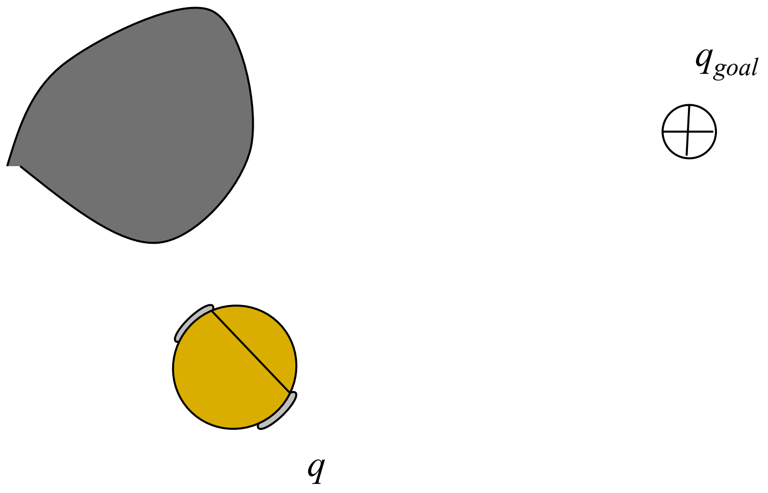
- In APFs, the robot is treated as a *point under the influence* of an artificial potential field.
  - Electrical analogy: The generated robot movement is similar to an electric charge under the force of an electric field
  - Mechanical analogy: The generated robot movement is similar to a ball rolling down the hill

Artificial Potential Fields

- In APFs
    - **Goals** generates **attractive** force
    - **Obstacles** generate **repulsive** forces

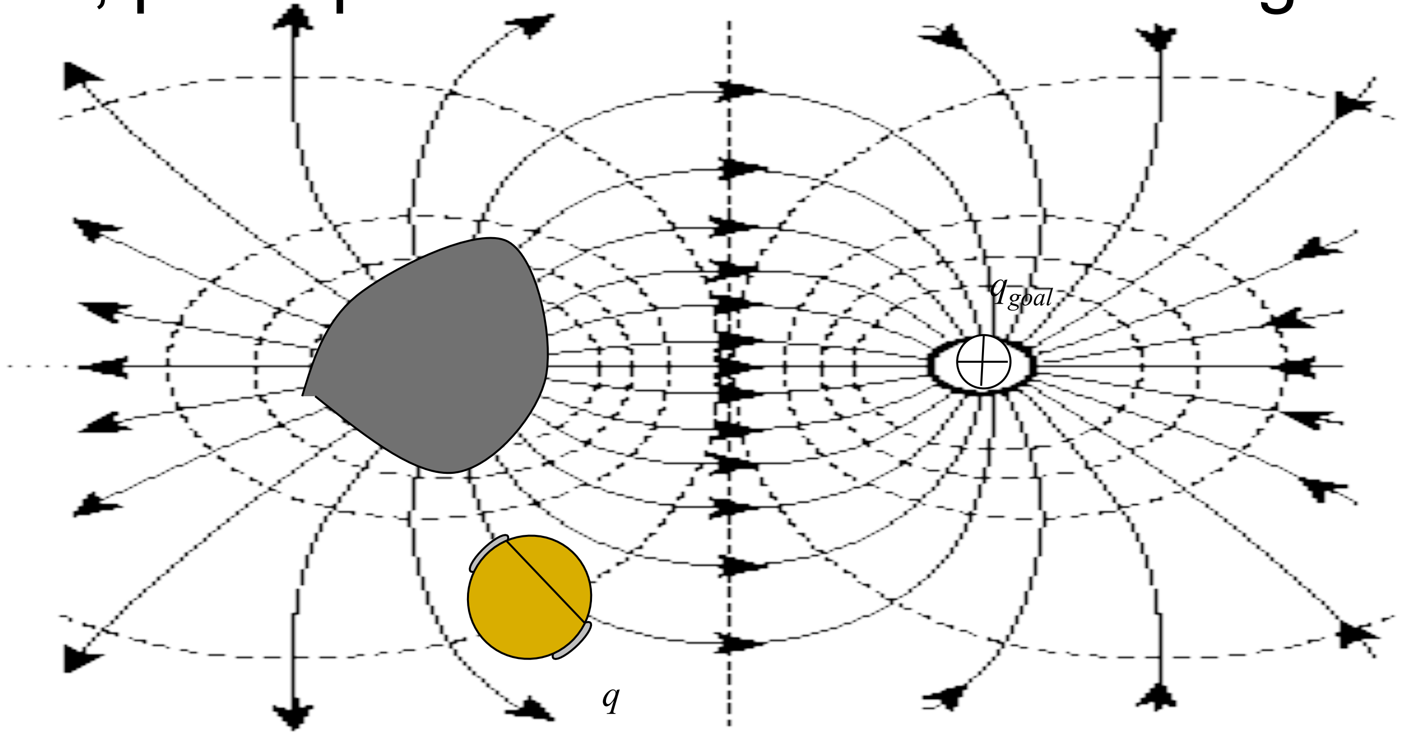
Artificial Potential Fields

- For a given configuration space and desired goal, place potentials on obstacles and goals



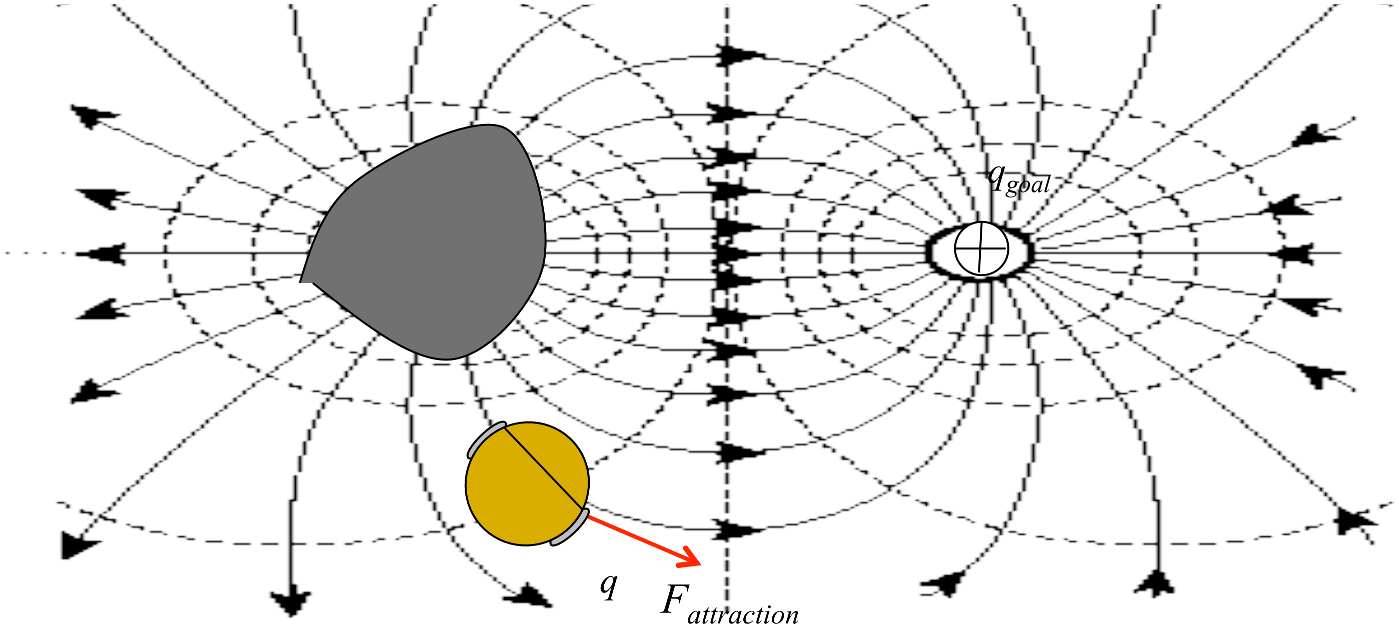
Artificial Potential Fields

- For a given configuration space and desired goal, place potentials on obstacles and goals

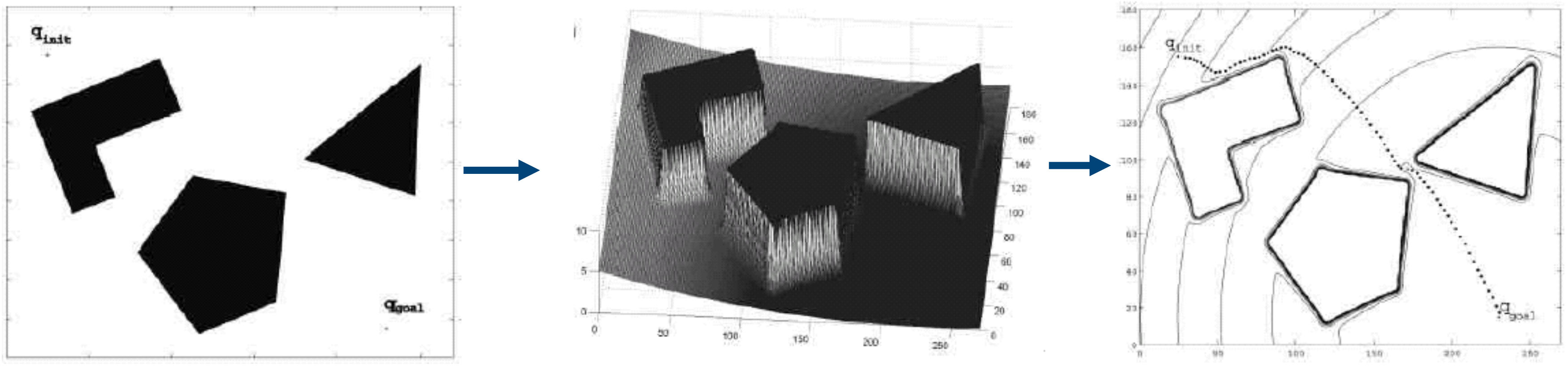


Artificial Potential Fields

- For any robot configuration q, the forces felt by the robot can be calculated to steer the robot towards the goal.



Artificial Potential Fields



Potential Field Generation

- Given potential functions U, Generate artificial force field $F(q)$
  - Sum all potentials (repulsive and attractive).
  - Differentiate to determine forces
  - Note: functions must be differentiable

$$\begin{aligned} F(q) &= -\nabla U(q) \\ &= -\nabla U_{att}(q) - \nabla U_{rep}(q) \\ &= \begin{bmatrix} -\delta U / \delta x \\ -\delta U / \delta y \end{bmatrix} \end{aligned}$$

Attractive Potential Fields

- Parabolic function representing the Euclidean distance $\rho_{goal}(q) = || q - q_{goal} ||$ to the goal.

$$U_{att}(q) = \frac{1}{2} k_{att} \rho^2_{goal}(q)$$

- Attracting force converges linearly towards 0 (goal)

$$\begin{aligned} F_{att}(q) &= -\nabla U_{att}(q) \\ &= -k_{att}(q - q_{goal}) \end{aligned}$$

Repulsive Potential Fields

- Generate a barrier around the obstacle
  - Does not influence robot if far from the obstacle

$$U_{rep}(q) = \begin{cases} \frac{1}{2} k_{rep} \left[\frac{1}{\rho(q)} - \frac{1}{\rho_0} \right]^2 & \text{if } \rho(q) \le \rho_0 \\ 0 & \text{if } \rho(q) > \rho_0 \end{cases}$$

  - Where $\rho(q) = || q - q_{obst} ||$ is the minimum distance to the object

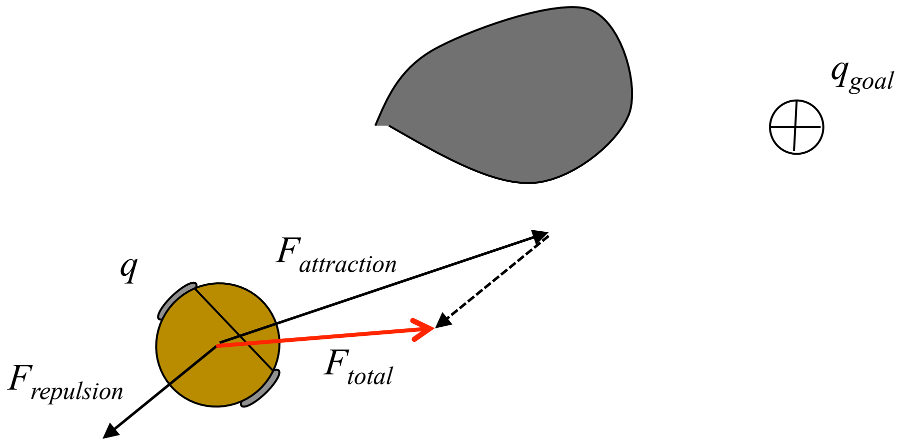
Repulsive Potential Fields

- Field is positive or zero and tends to infinity as q gets closer to the object

$$F_{rep}(q) = -\nabla U_{rep}(q)$$
$$= \begin{cases} k_{rep} \left(\frac{1}{\rho(q)} - \frac{1}{\rho_0} \right) \frac{q - q_{obst}}{\rho^3(q)} & \text{if } \rho(q) \le \rho_0 \\ 0 & \text{if } \rho(q) > \rho_0 \end{cases}$$

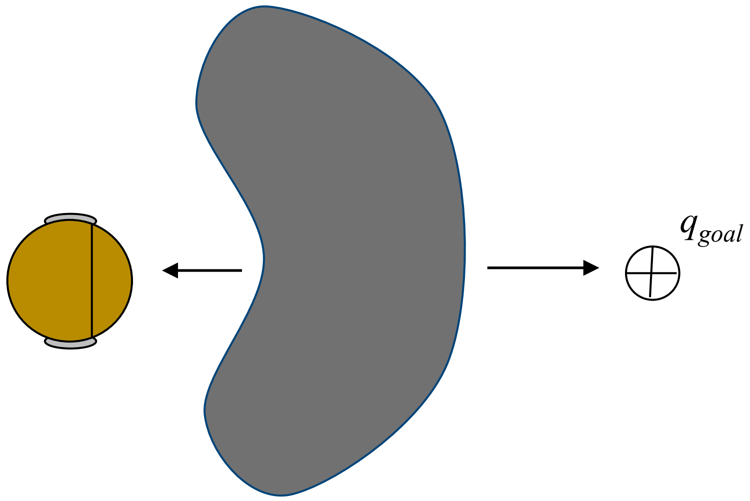
Artificial Potential Fields

- Given current configuration of the robot q
  1. **Sum** total force **vectors** $F(q)$ generated by the potential fields.
  2. Set desired robot velocity (v, w) proportional to the force $F(q)$



Artificial Potential Fields

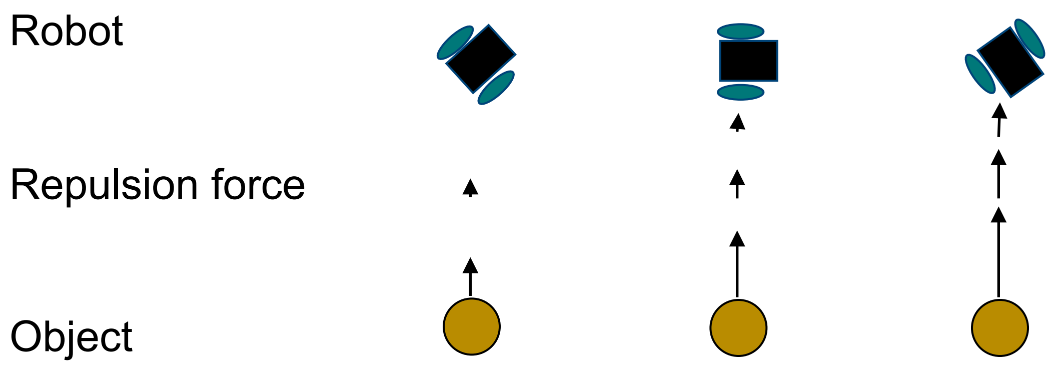
- Local minimums



  - If objects are not *convex* (i.e. concave), there exist situations where several minimal distances exist and can result in oscillations
  - Not complete

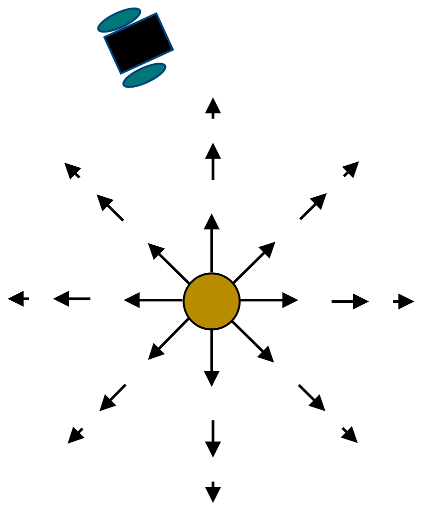
Artificial Potential Fields

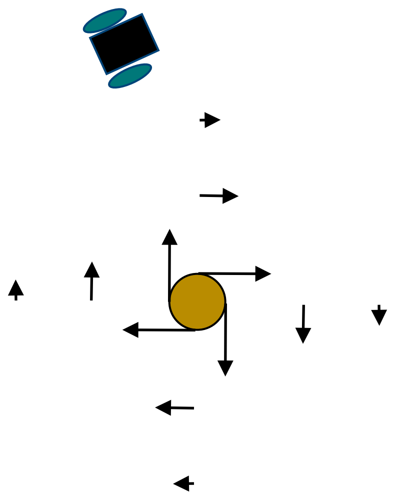
- Extended Potential Fields
  - Many modifications to potential fields have been done in order to improve completeness, optimality.
  - Example: Orientation based potentials
    - Can increase potential depending on orientation of robot



Artificial Potential Fields

- Extended Potential Fields
  - Also, can use rotational fields in one direction

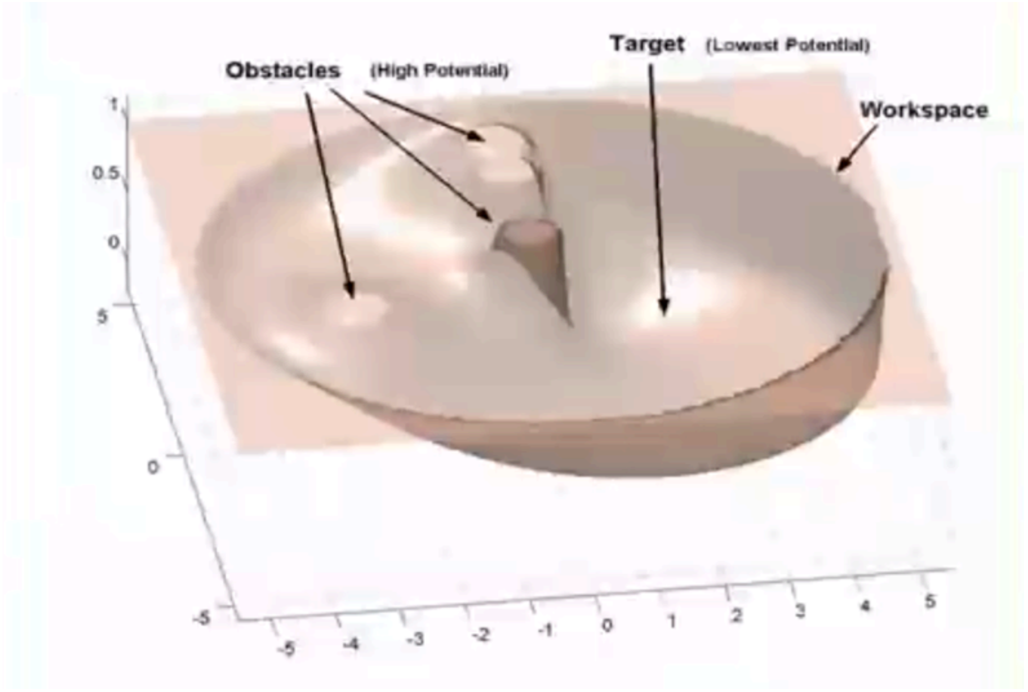

Linear source


Rotational source

Artificial Potential Fields

- Example:

  http://www.youtube.com/watch?v=r9FD7P76zJs


Target (Lowest Potential)
Obstacles (High Potential)
Workspace