0. SINGLE DOF KF

$z_1 = 2.50$ m

$z_2 = 3.00$ m

$\sigma^2 = 0.05 z^2$

$\Downarrow$

$\sigma_1^2 = 0.05(2.5)^2 =$

$\sigma_2^2 = 0.05(3.0)^2 =$

$$\hat{z} = z_1 + K(z_2 - z_1)$$

$$K = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} = \frac{0.05(2.5)^2}{0.05(2.5)^2 + 0.05(3.0)^2} = \frac{1}{1 + (1.20)^2} = 0.41$$

$$\sigma^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2} = 0.18$$

$$\hat{z} = 2.50 + (0.41)(3.0 - 2.5)$$

$$= 2.71$$

1. PROPAGATION OF ERRORS A

$\rho = 5.00 \ m \qquad \sigma_\rho^2 = 0.01 \ m^2$

$\alpha = \frac{30}{180}\pi \qquad \sigma_\alpha^2 = \frac{2}{(180)^2}\pi^2$

$x = \rho\cos\alpha$

$y = \rho\sin\alpha$

$\Downarrow$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \rho\cos\alpha \\ \rho\sin\alpha \end{bmatrix} = f(\rho,\alpha) \approx F_{\rho\alpha}\begin{bmatrix} \rho \\ \alpha \end{bmatrix}$$

$$F_{\rho\alpha} = \begin{bmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \alpha} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \alpha} \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha & -\rho\sin\alpha \\ \sin\alpha & \rho\cos\alpha \end{bmatrix}$$

$$\Sigma_{xy} = F_{\rho\alpha}\Sigma_{\rho\alpha}F_{\rho\alpha}^T$$

$$= \begin{bmatrix} \cos 30^\circ & -5\sin 30^\circ \\ \sin 30^\circ & 5\cos 30^\circ \end{bmatrix} \begin{bmatrix} 0.01 & 0 \\ 0 & 2\frac{\pi^2}{180^2} \end{bmatrix} \begin{bmatrix} \cos 30^\circ & \sin 30^\circ \\ -5\sin 30^\circ & 5\cos 30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.0113 & -0.0023 \\ -0.0023 & 0.0139 \end{bmatrix}$$

2. PROPAGATION OF ERRORS B

$\Delta t = 0.1$

ASSUME:
$b = 0.1$

$$u_t = \begin{bmatrix} \Delta S_R \\ \Delta S_L \end{bmatrix} = \begin{bmatrix} 0.010 \\ 0.010 \end{bmatrix} \qquad \Sigma_u = \begin{bmatrix} 0.1|\Delta S_R| & 0 \\ 0 & 0.1|\Delta S_L| \end{bmatrix}$$

$$x_{t-1} = \begin{bmatrix} 1.00 \\ 1.00 \\ 0.00 \end{bmatrix} \qquad P_{t-1} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$x'_t = \begin{bmatrix} x'_t \\ y'_t \\ \theta'_t \end{bmatrix} = f(x_{t-1}, u_t) = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(\Delta S_L + \Delta S_R)\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) \\ \frac{1}{2}(\Delta S_L + \Delta S_R)\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) \\ \frac{\Delta S_R - \Delta S_L}{b} \end{bmatrix}$$

$$\Downarrow$$

$$x'_t = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(0.01+0.01)\cos(0) \\ \frac{1}{2}(0.01+0.01)\sin(0) \\ 0 \end{bmatrix} = \begin{bmatrix} 1.01 \\ 1.0 \\ 0 \end{bmatrix}$$

$$P'_t = F_{x_{t-1}} P_{t-1} F_{x_{t-1}}^T + F_u \Sigma_u F_u^T$$

$$F_{x_{t-1}} = \begin{bmatrix} \frac{\partial x'_t}{\partial x_{t-1}} & \frac{\partial x'_t}{\partial y_{t-1}} & \frac{\partial x'_t}{\partial \theta_{t-1}} \\ \frac{\partial y'_t}{\partial x_{t-1}} & \frac{\partial y'_t}{\partial y_{t-1}} & \frac{\partial y'_t}{\partial \theta_{t-1}} \\ \frac{\partial \theta'_t}{\partial x_{t-1}} & \frac{\partial \theta'_t}{\partial y_{t-1}} & \frac{\partial \theta'_t}{\partial \theta_{t-1}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\frac{1}{2}(\Delta S_L + \Delta S_R)\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) \\ 0 & 1 & \frac{1}{2}(\Delta S_L + \Delta S_R)\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_u = \begin{bmatrix} \dfrac{\partial x_t'}{\partial \Delta S_R} & \dfrac{\partial x_t'}{\partial \Delta S_L} \\[2ex] \dfrac{\partial y_t'}{\partial \Delta S_R} & \dfrac{\partial y_t'}{\partial \Delta S_L} \\[2ex] \dfrac{\partial \theta_t'}{\partial \Delta S_R} & \dfrac{\partial \theta_t'}{\partial \Delta S_L} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) - \frac{1}{2}(\Delta S_L + \Delta S_R)\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right)\left(\frac{1}{2b}\right) & \frac{1}{2}\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) - \frac{1}{2}(\Delta S_L + \Delta S_R)\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right)\left(\frac{-1}{2b}\right) \\[2ex] \frac{1}{2}\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) + \frac{1}{2}(\Delta S_L + \Delta S_R)\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right)\left(\frac{1}{2b}\right) & \frac{1}{2}\sin\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right) + \frac{1}{2}(\Delta S_R + \Delta S_L)\cos\left(\theta_{t-1} + \frac{\Delta S_R - \Delta S_L}{2b}\right)\left(\frac{-1}{2b}\right) \\[2ex] \frac{1}{b} & \frac{-1}{b} \end{bmatrix}$$

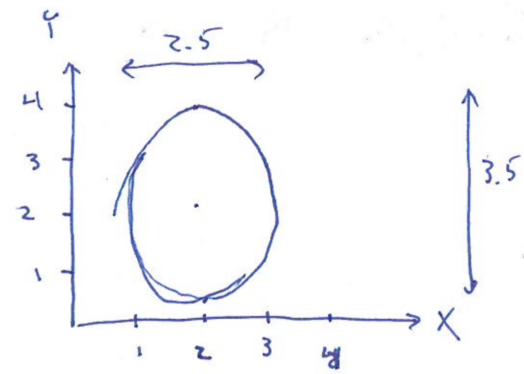
$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0.05 & -0.05 \\ 10 & -10 \end{bmatrix}$$

$$P_t' = F_{x,t-1} P_{t-1} F_{x,t-1}^T + F_u \Sigma_u F_u^T$$

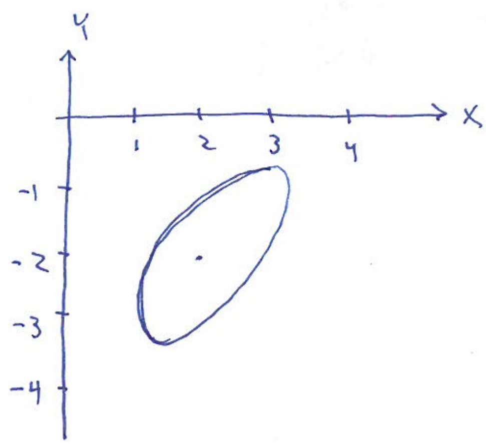
$$= \begin{bmatrix} 0.2005 & 0.0001 & 0 \\ 0.0001 & 0.2000 & 0.001 \\ 0 & 0.001 & 0.3 \end{bmatrix}$$

3. CONFIDENCE ELLIPSE

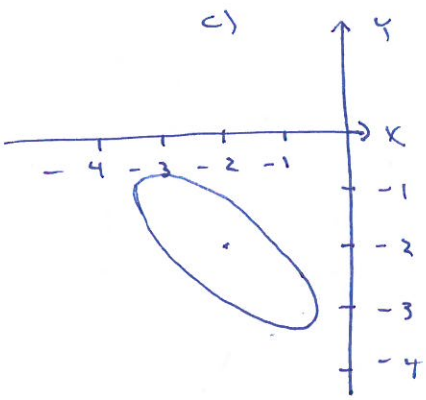
a)



b)



c)



4. INNOVATION

GIVEN:

$$P_t, \quad z_{exp} = h^i(x_t^i, M), \quad R_t, \quad z_t$$

$$\Sigma_{IN} = H P_t H^T + R$$

$$H = \left[\frac{\partial x_t^i}{\partial z^i} \quad \frac{\partial y_t^i}{\partial z^i} \quad \frac{\partial \theta_t^i}{\partial z^i} \right]$$