The topic for today's class is analog electronics. Electrical systems are broken broadly into two major classes: digital electronics and analog electronics.

We have talked quite a bit about digital electronics, including elements uses in digital circuits such as AND, OR and XOR gates.

The digital electronics we have talked about have use *binary* 1s and 0s to carry information. But it would be more proper to say that digital electronics are any system that use quantized states to encode information. If each element of the digital system only encodes two states than it is a binary digital system, which is the most common type.

What is a 1 or a 0 though? How does a circuit or system "know"?

To have an encoding for 1 or 0, our system must rest upon some physical parameter. While we could create a digital system based on *any* physical parameter the most convent set of parameter to exploit are electrical ones. This is the domain of analog electronics

What is electricity?

Electricity can be defined a number of ways, but is the result of the electrical forces between charged particles. The physics of these rules is stated completely via 4 equations known as Maxwell's equations.

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$
$$\nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t}$$
$$\nabla \times B = \mu_0 (J + \frac{\varepsilon_0 \partial E}{\partial t})$$

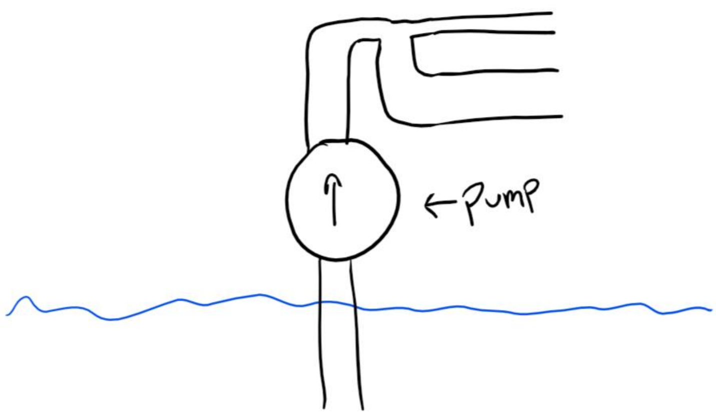
A full discussion of Maxwell's equations is a topic for 1 to many semesters of physics. Generally speaking Maxwell's equations talk about electrical interactions as a form of wave mechanics.

Maxwell wrote these equations down in the 1861. This period of history marked the formalization of the theory of electricity and magnetism. Before this time people had other theories of electricity, while these theories are wrong in sometimes significant ways, they were theories because they explained what was observed up until that time. Old theories that are known to be incorrect can still be useful. Newton's laws are not correct, relativity is needed, yet for slow objects with large masses Newton's laws work just fine.

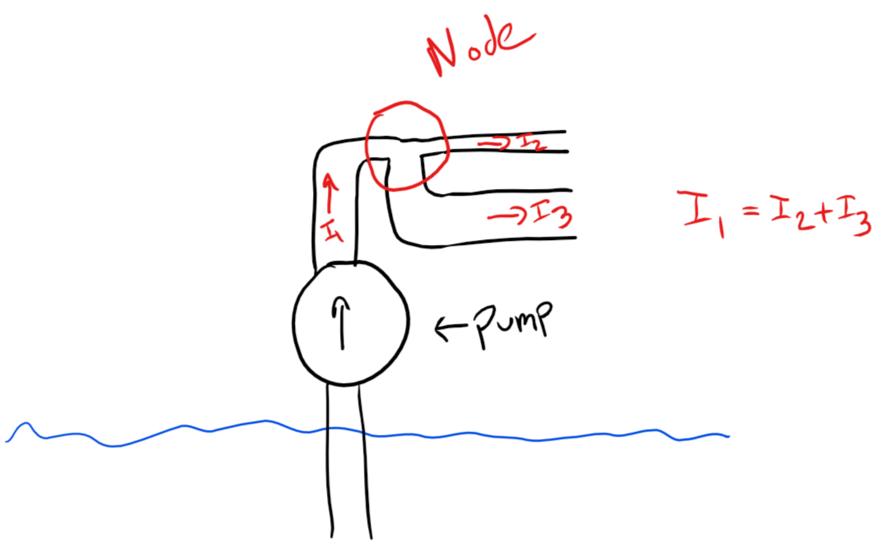
The same is true for electricity. One of the earlier models of electricity was that it was a fluid.

Let's for the moment assume that electricity is some fluid similar to water to gain some intuition on how electricity should work.

Let's consider the following system:




Here I have a pump attached to a pipe submerged into a large pool of water. It flows a constant amount of water up through it. Its output is attached to a pipe that splits into two pipes of different diameters. Consider the behavior of the water as it flows through the system.




We may define the amount of fluid flowing in the pipe as a current: $I$.

If we make this definition then we may observe that in steady state the amount of current going a point where multiple branches join is 0. The amount in is exactly equal to the amount out. If the amount exceeds this the pressure at the point will increase which will increase the flow out.

In analog circuits the current flow is measured in Amperes (Amps), which is Coulombs/second. An electron has a charge of 1.602e-19 Coulombs so an Amp is a measure of how many electrons are flowing by per second, the flow of our electronic fluid.

If we were to write the observation in a more mathematical way we would say, at any node

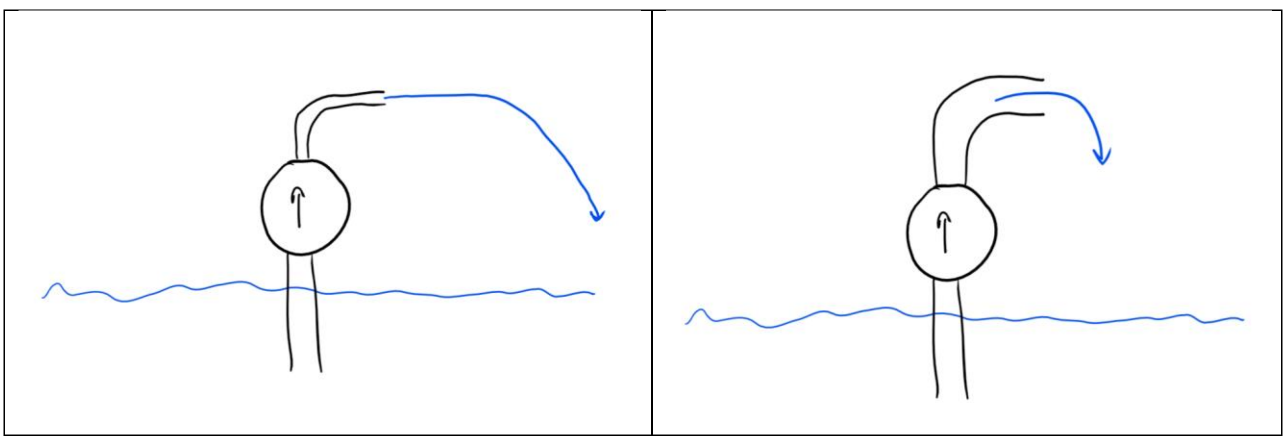
$$\sum_n I_n = 0$$

This is called Kirchhoff's current law or KCL.

The other observation we can make about our pipe circuit is that the amount of fluid coming out of the large pipe should be larger than the small pipe. Indeed we would expect that the flow would be directly proportional to the cross sectional area of the pipe. We will return to this observation in a moment.

A fluid can have a flow rate a current, which in analog circuits we call $I$. But it can also have a pressure. Which we call $V$, this is the voltage (measured in Volts).

To see what a voltage is physically again consider our pump system but with only 1 pipe attached:



In both cases the same amount of fluid must flow out per time. Given the smaller pipe has a smaller cross section the velocity of the fluid coming out must be greater (if the density of the fluid is constant). To create a greater velocity we need a larger *differential* pressure. Voltages which are like pressure is only meaningful as a differential across something, in this case it is a pipe.

We can observe from this thought experiment that the pipe size has something to do with relating the pressure to the flow. It has something to do with relating the voltage to the current.

This parameter that relates voltage to current is called: impedance, which we give a symbol Z to.

The relationship between voltage and current is called Ohm’s law:

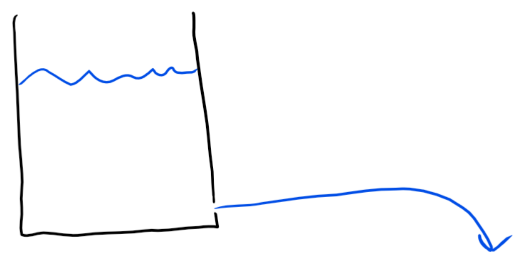
$$V = IZ$$

Sometimes you will see this written as:

$$V = IR$$

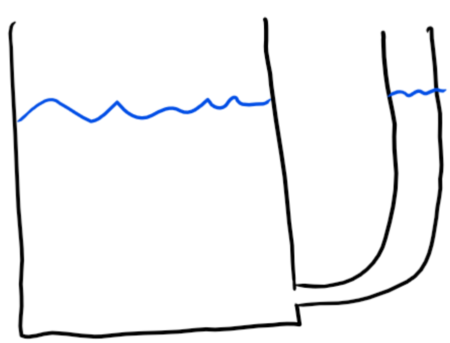
Where $R$ is called the resistance. A resistance is just a specific type of impedance. The Z form is more general.

Let's consider one fluid thought experiment:



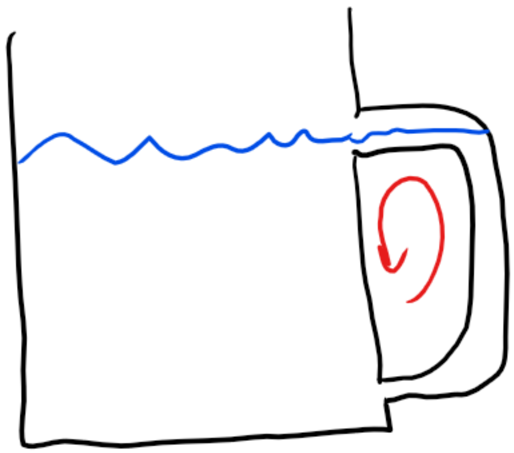
If I have a large tank of water and I put a small hole in the bottom of it the water will shoot out because there is some pressure at the bottom of the tank due to the mass of the water and gravity.

Now if I modify the tank to have a pipe on the output that bends upwards:



Then we would observe that the level in the pipe rises to be exactly level with the tank. The pipe pointed up *drops* all of the pressure from the bottom of the tank. So this pipe which again remember is an called an impedance drops all of the pressure. The pressure difference between the top of the pipe and the tank is 0. The voltage difference is 0.

Circuits have to be connected in loops for current to flow because of KCL so let me modify this a little to be a loop.



The voltage around the *loop* shown in red is 0. This observation is called Kirchhoff's voltage law (KVL).

Formally stated it says that:

$$\sum_n V_n = 0$$

The sum of the voltages around any loop in a circuit is 0.

Let's recap, we now know 3 laws of electricity: KVL, KCL and Ohm's law.

KCL says that the current into and out of any node of a circuit is 0. ($\sum_n I_n = 0$)

KVL says that the voltages around any loop in a circuit is 0. ($\sum_n V_n = 0$)

Ohm's law says that the ratio of the voltage drop across an element to the current in the element is the impedance $\frac{V}{I} = Z$. ($V = IZ$)

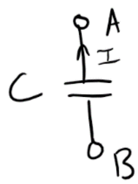
There are three types of ideal linear passive circuit elements: the resistor, the capacitor and the inductor.

These have the following symbols and terminal relationships:


Resistor

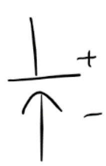
$$V_A - V_B = I R$$

$$V = IR$$

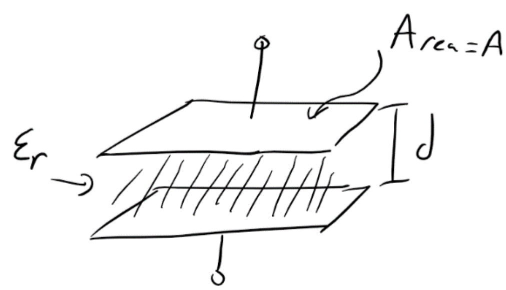



Capacitor

$$V = I Z_C = \frac{I}{j\omega C} = \frac{I}{sC}$$



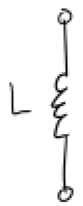

Polarize capacitor




$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{\varepsilon A}{d}$$

Parallel plate approximation

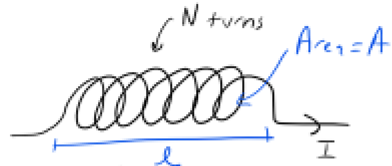
Farads $\rightarrow$ F




Inductor

$V = sLI = j\omega LI$

$\Phi = LI$




Solenoid

$\Phi = \mu \frac{N^2 I A}{l}$

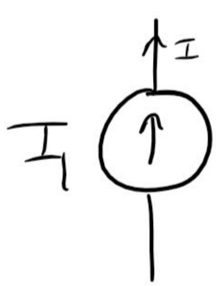
$L = \mu \frac{N^2 A}{l}$

Henry $\rightarrow$ H

Note that the impedance of an inductor, capacitor and resistor are:

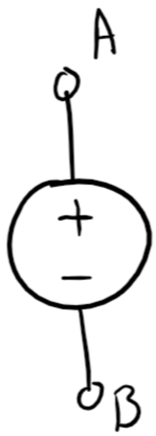
$$Z_R = R$$
$$Z_c = \frac{1}{j\omega c}$$
$$Z_L = j\omega L$$

We have two types of sources of power for circuits, current sources which supply a constant current like the pump did in our examples and voltage sources:




General current source

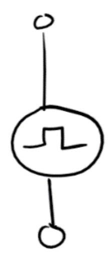
$I = I_1 = 1\,A$ or $10\sin(\omega t)$ . . .

$V_1 = y$ 

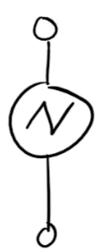
General Voltage source

$$V_A - V_B = y$$

You will also see voltage sources with functions drawn inside:

$V_1 = f_n$  or 

or

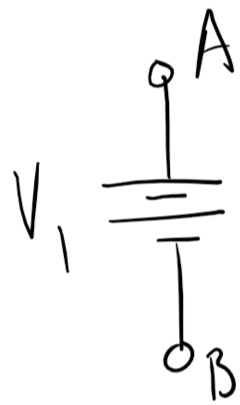
 ....

Function Voltage Source

$V_A - V_B = f_n$

i.e. $V_0 \sin(wt)$

And DC voltage sources often drawn just as batteries:




DC Voltage

or

Battery (2 cells = shown)

$$V_A - V_B = V_1$$

Finally because voltages are differences really, we need a zero point. This is called ground, for which there are several symbols:

Ground

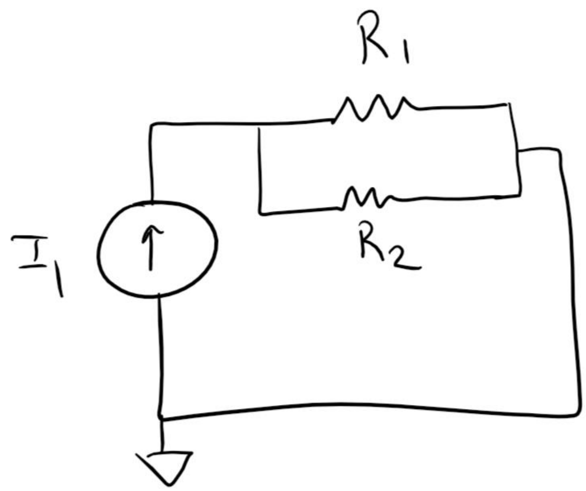
 or  Signal GND

 Earth GND

 Chassis GND

Let's get started looking at circuits. As the resistor is the easiest passive element to deal with we will start with just resistors and power supplies.

Consider the following circuit:




This circuit has two nodes and two loops.

This is the circuit equivlent to the first water thought experiement. How much current flows through R1?

We can use KCL to write down that:

$$I_1 - I_{R1} - I_{R2} = 0$$

We can use KVL to write down that:

$$V_{I1} - V_{R2} = V_{I1} - V_{R1} = 0$$
$$V_{R2} = V_{R1}$$

This second equation also comes from the fact that the two resistors are in *parallel*. We stay that two elements are in parallel when they share both terminals.

We can permute this second equation into one about currents by using the terminal relationship of a resistor:

$$V = IR$$

So:

$$I_{R2}R_2 = I_{R1}R_1$$

Substuting this into the equation from KCL we find:

$$I_1 = I_{R1} + \frac{I_{R1}R_1}{R_2} = \frac{I_{R1}(R_2 + R_1)}{R_2}$$

Solving for the desired current:

$$I_{R1} = \frac{I_1 R_2}{R_1 + R_2}$$

This configuration is so common that it is given a name: the current divider rule. It basically says the bigger the other pipe (the lower the other pipe's resistance) the less fluid will flow in the pipe in parallel with it.

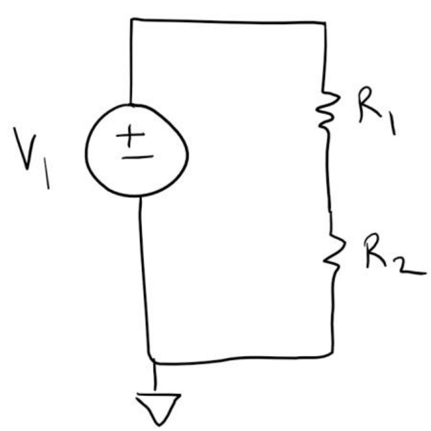
We can generalize this rule to write:

$$I_{Z1} = \frac{I_1 Z_2}{Z_1 + Z_2}$$

Where the two elements in parallel are now any impedance, rather than just a resistor.

Elements can also be connected end-to-end. This type of connection is called a series connection. In a series connection there is only one path for the electricity to flow along (rather than multiple).

Let's look at a simple series circuit:




This circuit has 3 nodes and 1 loop. How much voltage is across R1?

We can use KCL at each node.

$$I_{V1} = I_{R1}$$
$$I_{R1} = I_{R2}$$
$$I_{R2} = I_{V1}$$

In total this says what we already observed, there is only one path for the current to flow therefore all of the elements share the same current.

KVL says that:

$$V_1 - V_{R1} - V_{R2} = 0$$

From the terminal relationships we know that:

$$V_{R1} = I_{R1}R_1$$

$$V_{R2} = I_{R2}R_2 = I_{R1}R_2 = \frac{V_{R1}}{R_1}R_2$$

Substituting this into our KVL equation we find:

$$V_1 = V_{R1} + \frac{V_{R1}R_2}{R_1} = V_{R1}\left(1 + \frac{R_2}{R_1}\right) = \frac{V_{R1}(R_1 + R_2)}{R_1}$$

Solving for the desired voltage:

$$V_{R1} = \frac{V_1 R_1}{R_1 + R_2}$$

This type of circuit is also common so it has a special name: the voltage divider rule.

What is the current in the circuit?

What is the voltage drop across R2?

Like the current divider we can generalize this to any impedance:

$$V_{Z1} = \frac{V_1 Z_1}{Z_1 + Z_2}$$