ARW – Lecture 03
Motion Planning

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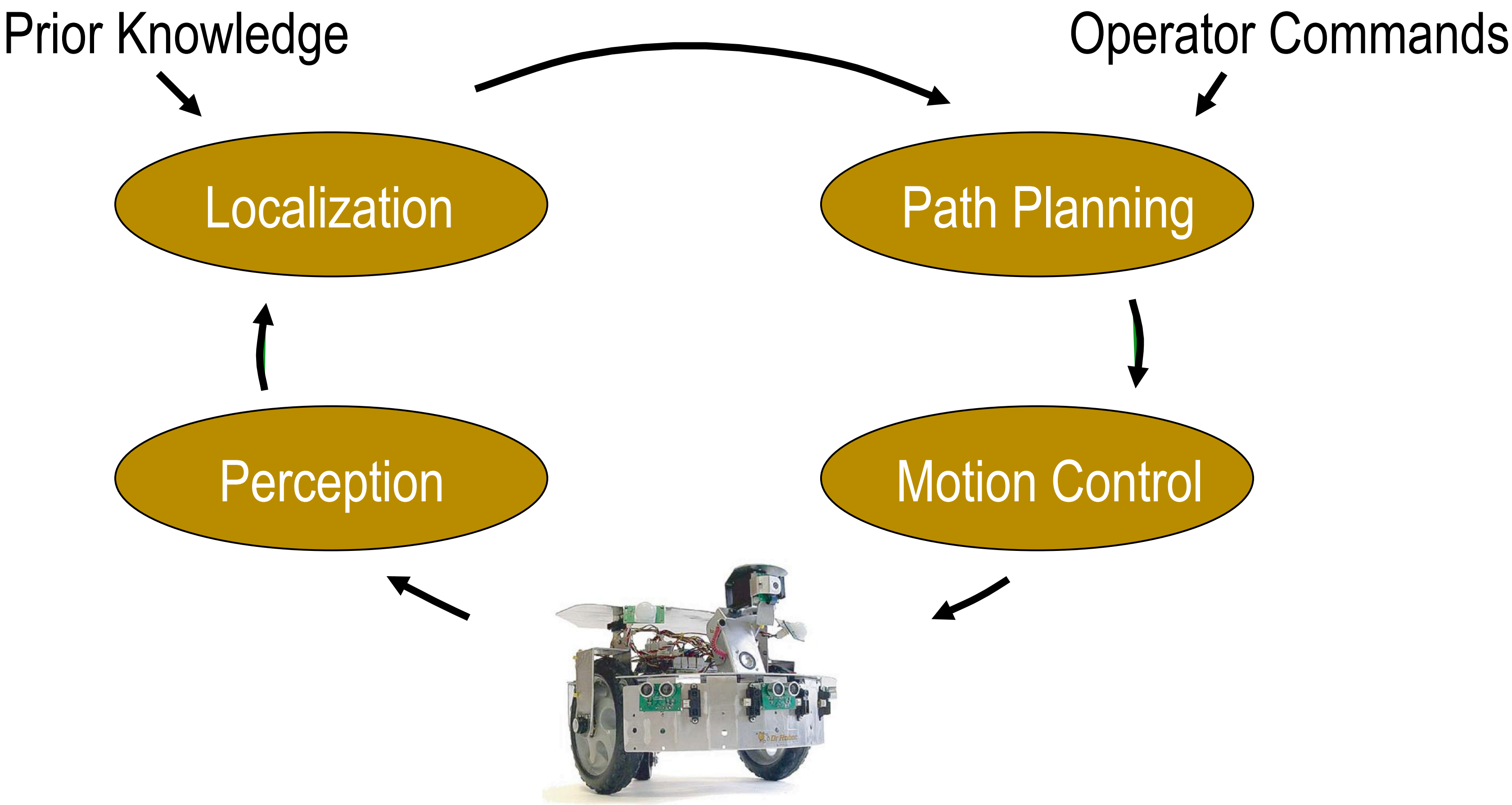
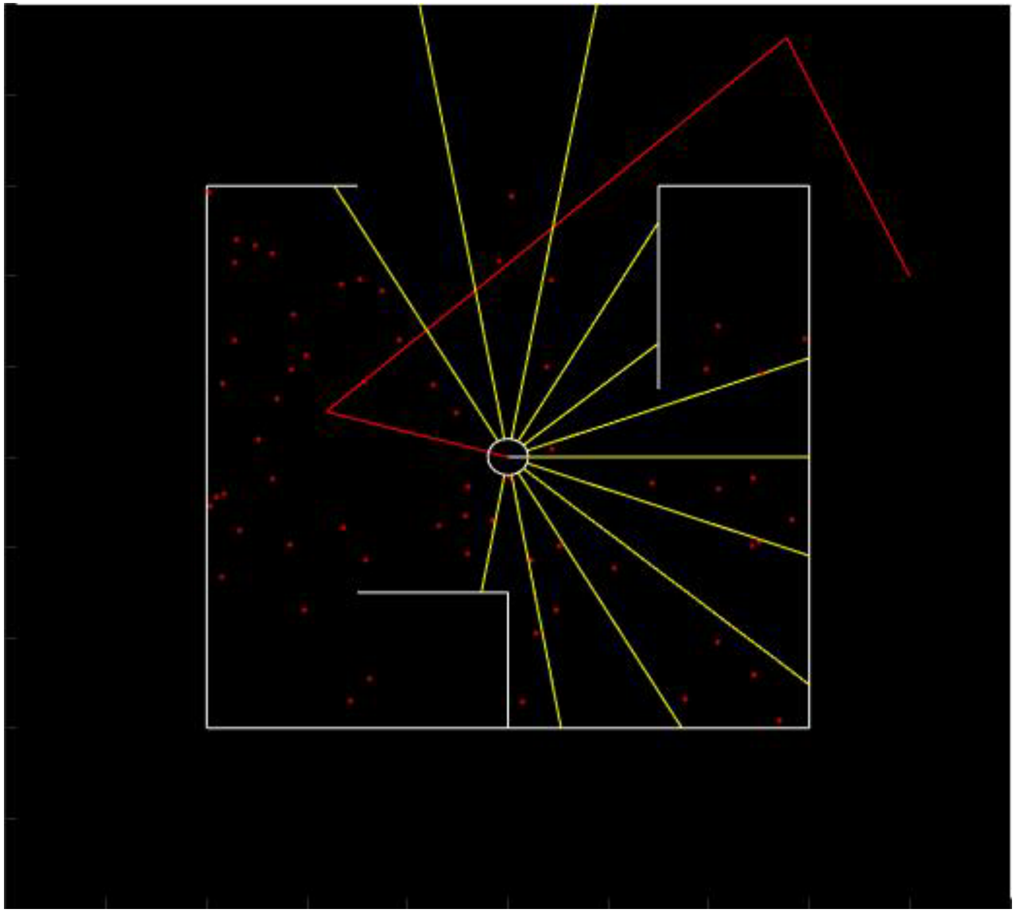
Semester: Summer 2016

Figures courtesy of Siegwart & Nourbakhsh

Planning Based Control



ARW Goals



Odometry Kinematics

- Lecture Goal

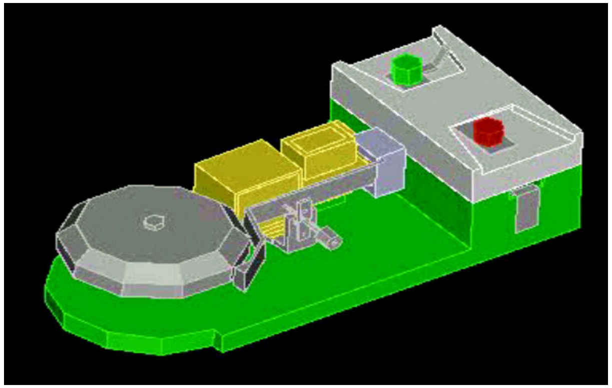
  - Develop a collision-free path given an initial robot state, a goal robot state, and a map.

$$T = f(X_0, X_G, M)$$

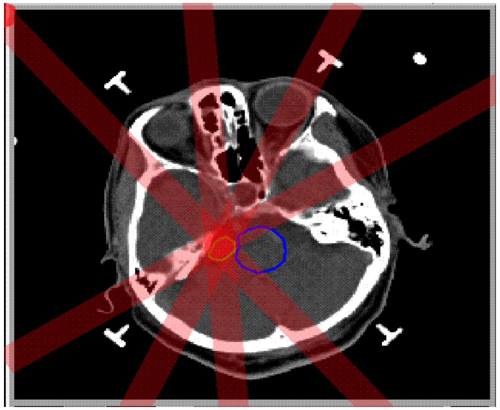
Introduction to Motion Planning

1. MP Overview
2. The Configuration Space
3. General Approach to MP
4. Metrics
5. PRMs
6. Single Query PRMs

MP Overview

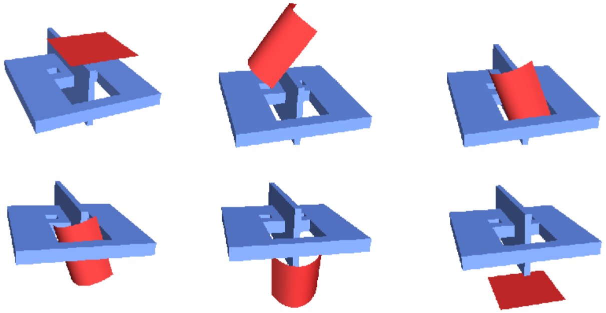


Assembly Planning, Latombe



Cross-Firing of a Tumor, Latombe

Tomb Raider 3 (Eidos Interactive)



Deformable Objects, Kavraki

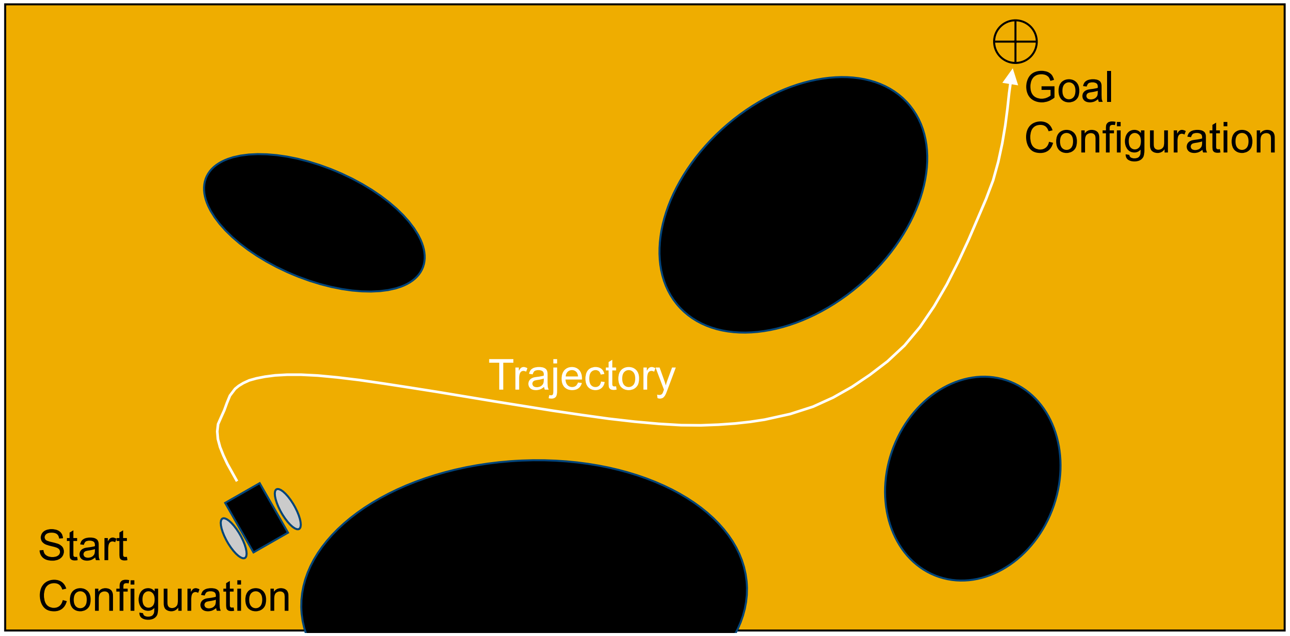
MP Overview

- Goal of robot motion planning:

To construct a collision-free path from some initial configuration to some goal configuration for a robot within a workspace containing obstacles.

MP Overview

- Example:



MP Overview

- Inputs
  - Geometry of robots and obstacles
  - Kinematics/Dynamics of robots
  - Start and Goal configurations

- Outputs
  - Continuous sequence of configurations connecting the start and goal configurations

Introduction to Motion Planning

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The Configuration Space

- To facilitate motion planning, the **configuration space** was defined as a tool that can be used with planning algorithms.

(Latombe 1991)

The Configuration Space

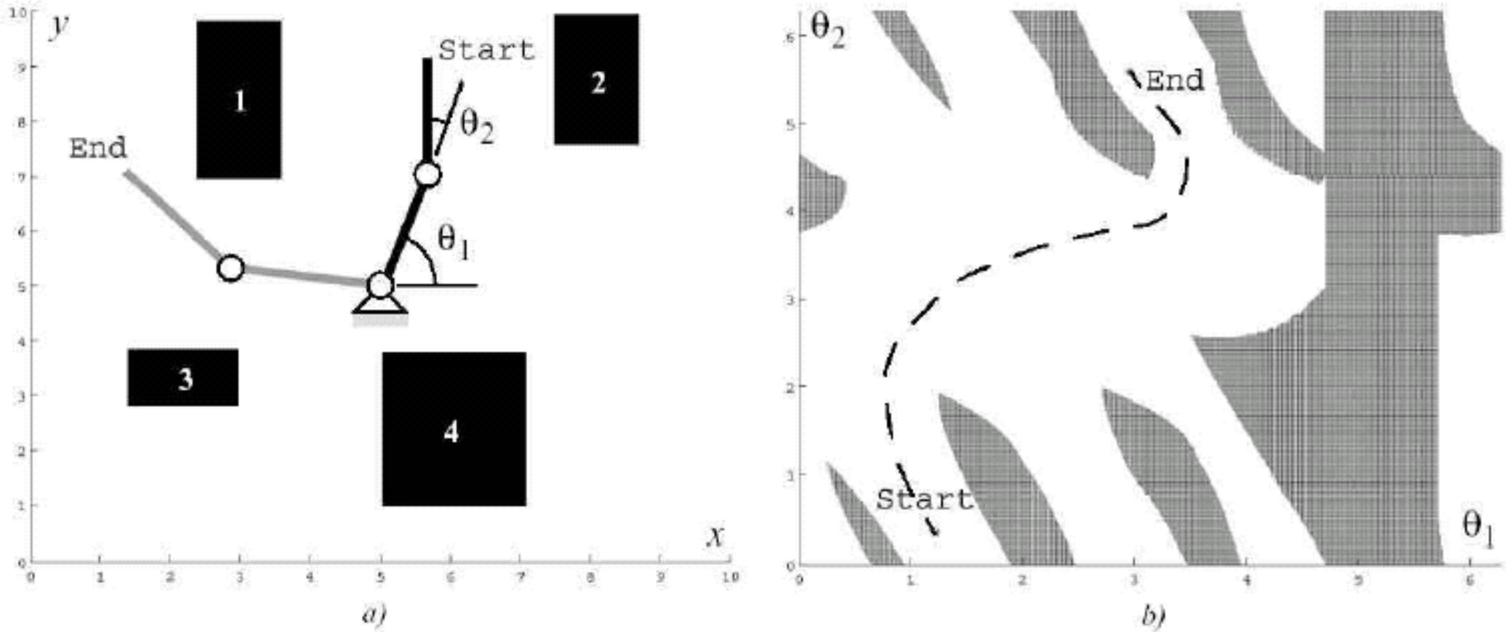
- A configuration q will completely define the state of a robot (e.g. mobile robot x, y, θ)
- The configuration space C, is the space of all possible configurations of the robot.
- The free space $F \subseteq C$, is the portion of the free space which is collision-free.

The Configuration Space

- The goal of motion planning then, is to find a path in F that connects the initial configuration q_{start} to the goal configuration q_{goal}

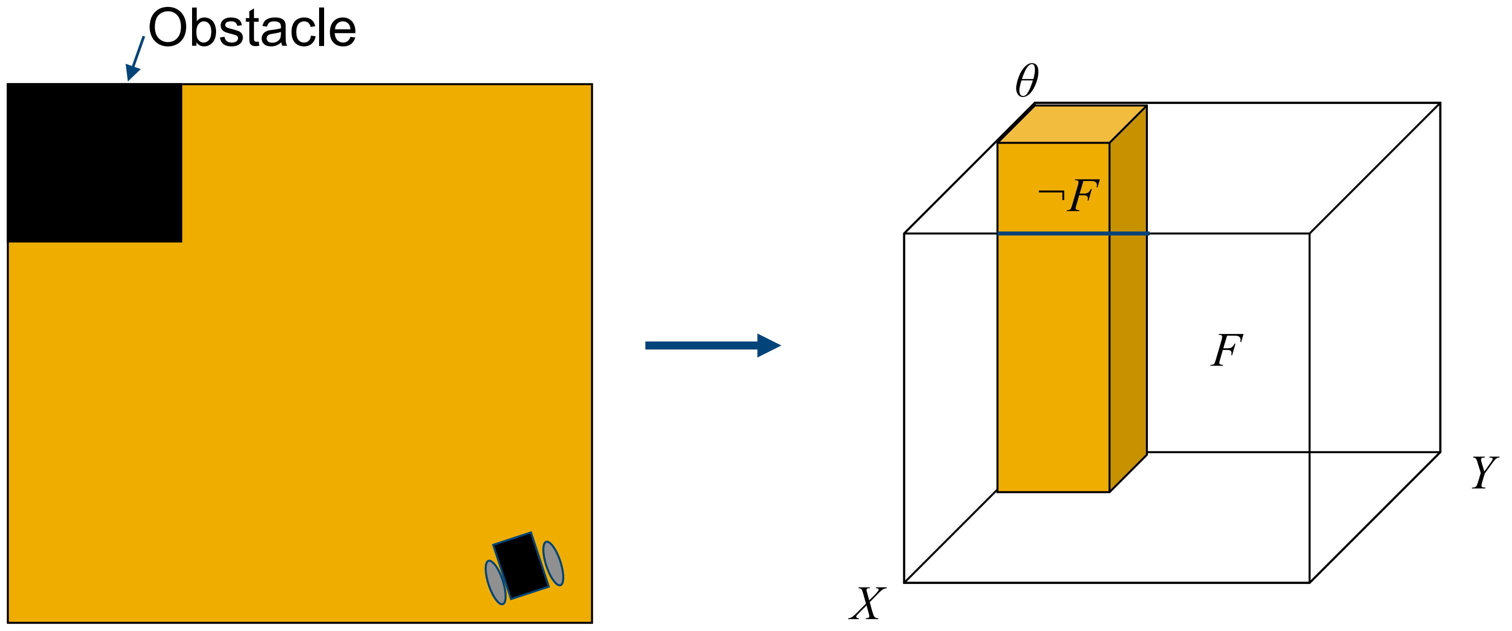
The Configuration Space

- Example 1: 2DOF manipulator:



The Configuration Space

- Example 2: Mobile Robot

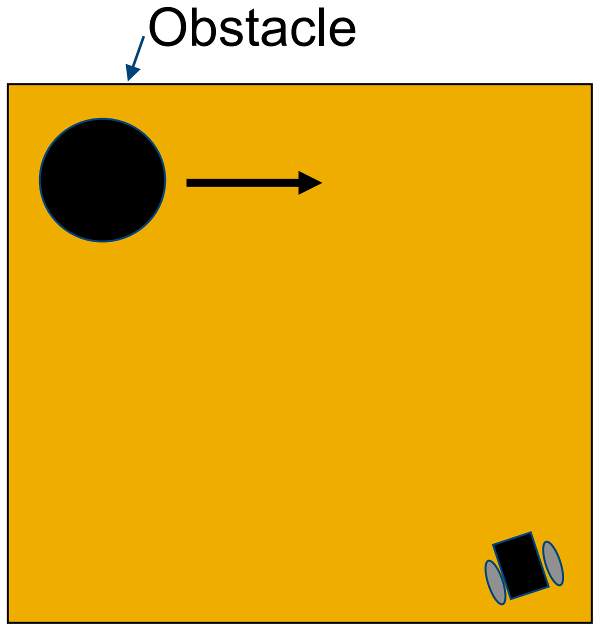


Workspace

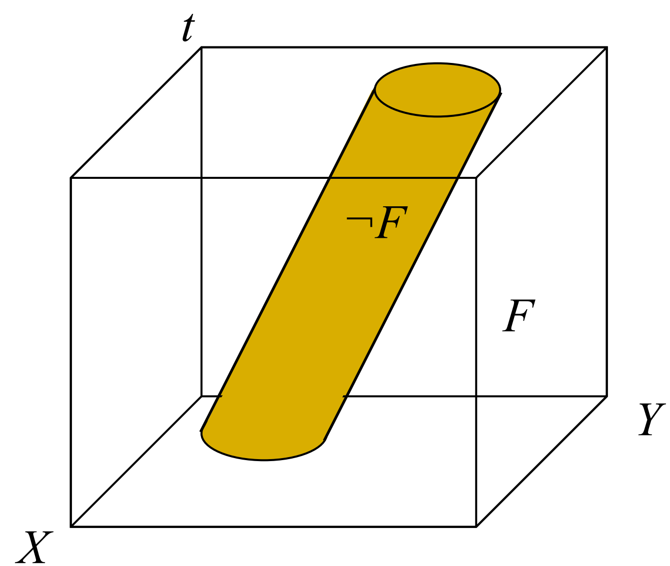
Configuration Space

The Configuration Space

- Example 3: Mobile Robot with moving obstacle



Workspace



Configuration Space

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General Approach to MP

- Motion planning is usually done with three steps:

  1. Define C
  2. Discretize C
  3. Search C

1. Define C

- Each planning problem may have a different definition of C.
  - Example 1: Include 3DOF for a mobile robot in static environment - (x,y,θ).
  - Example 2: Include only 2DOF for a mobile robot in static environment - (x,y).
  - Example 3: Include 5DOF for a mobile robot in dynamic environment - (x,y,θ,v,t).

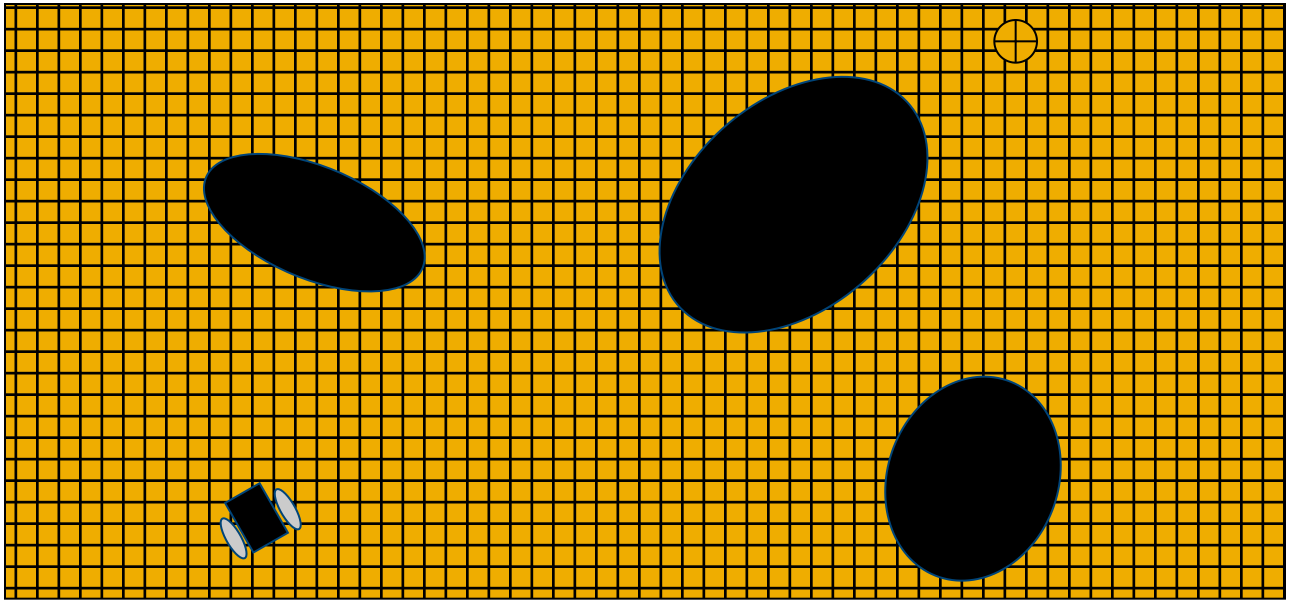
2. Discretize *C*

- Typical Discretizations:

  1. Cell decomposition
  2. Roadmap
  3. Potential field

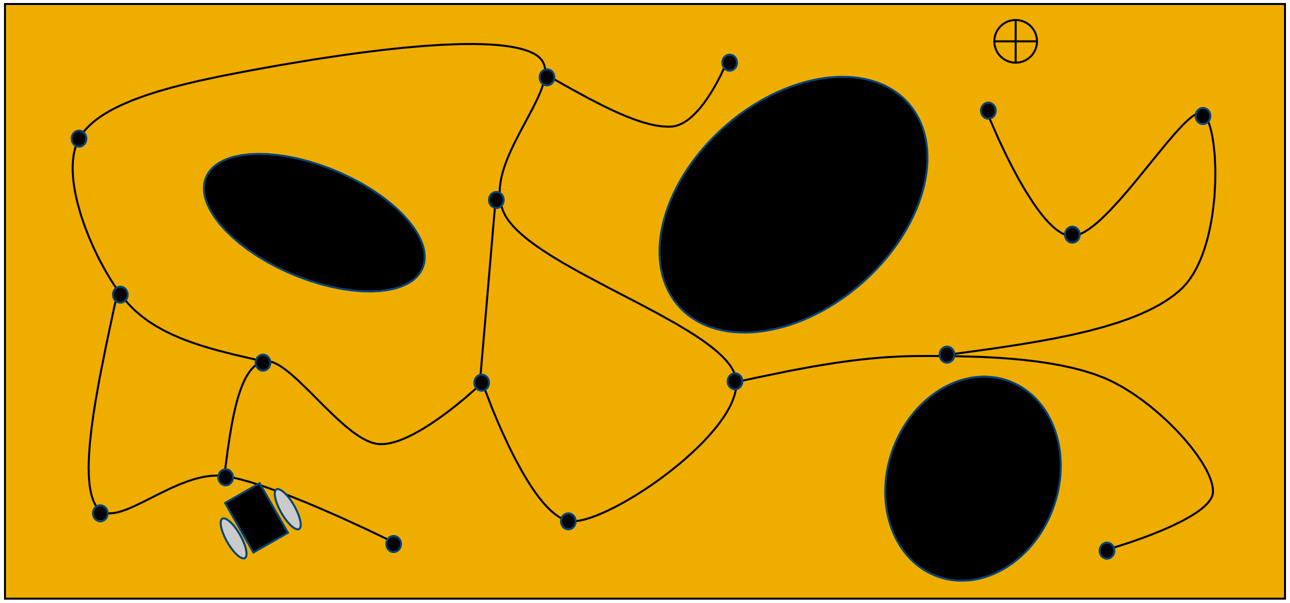
2. Discretize *C*

- Cell decomposition
  - Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells



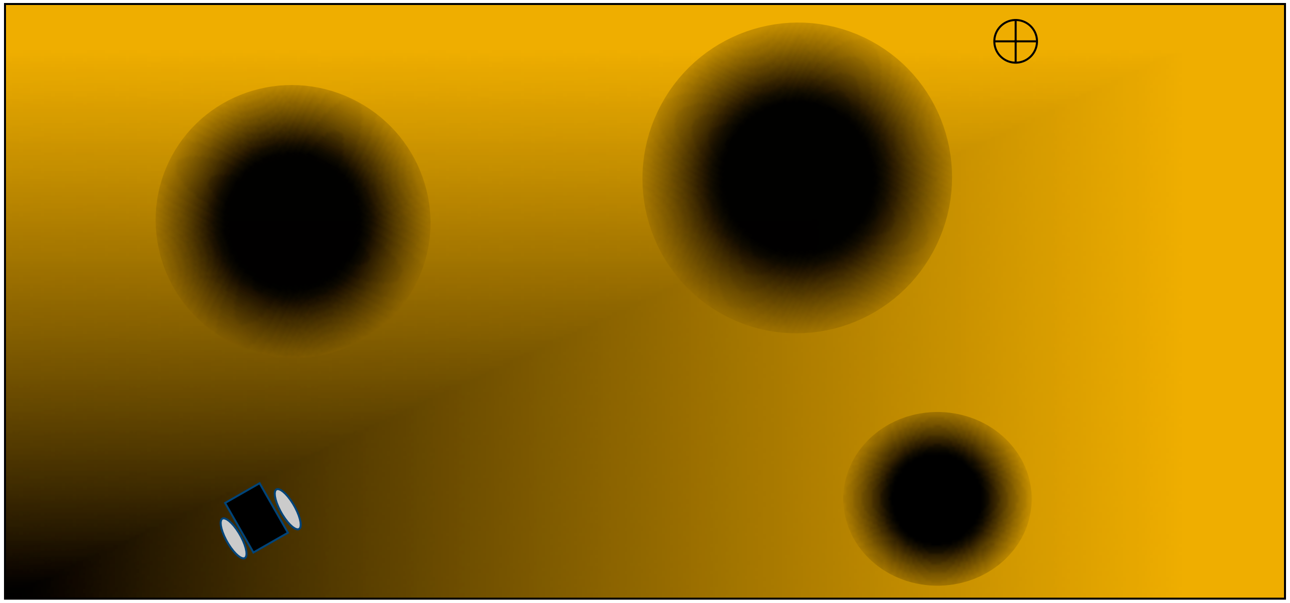
2. Discretize *C*

- Roadmap
  - Represent the connectivity of the free space by a network of 1-D curves



2. Discretize *C*

- Potential field
  - Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent



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Metrics

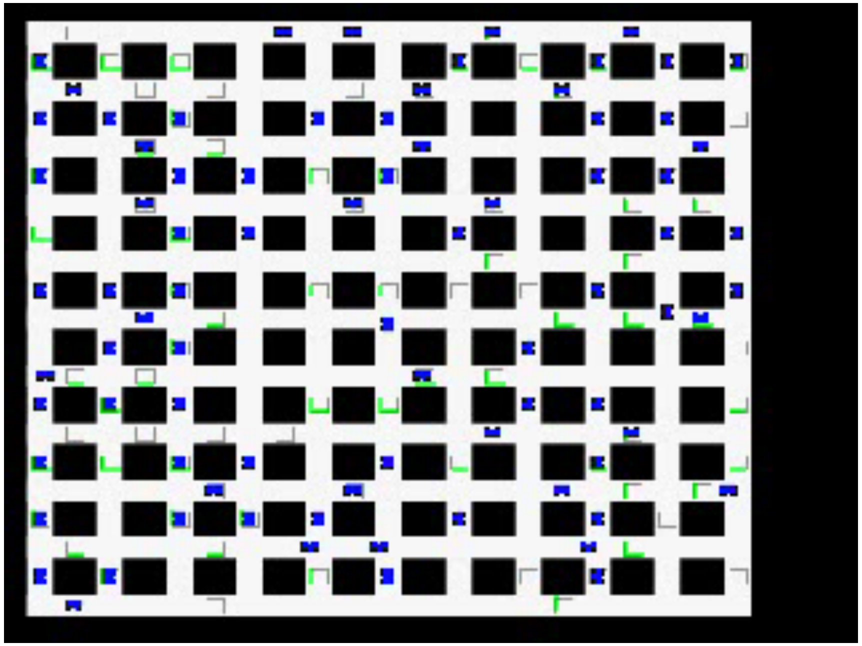
- Metrics for which to compare planning algorithms:
  1. Speed or Complexity
  2. Completeness
  3. Optimality
  4. Feasibility of solutions

Metrics

- We are left with…
  - Theoretical algorithms
    - Strive for completeness and minimal worst-case complexity
    - Difficult to implement
  - Heuristic algorithms
    - Strive for efficiency in common situations
    - Use simplifying assumptions
    - Weaker completeness
    - Exponential algorithms that work in practice

Motion Planning: Searching the Configuration Space

- Example: Multi Robot MP



Introduction to Motion Planning

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5. PRMs
6. Single Query PRMs

Probabilistic Road Maps

- Definition:
  - A probabilistic road map is a discrete representation of a continuous configuration space generated by randomly sampling the free configurations of the C-space and connecting those points into a graph.

Probabilistic Road Maps

- Goal of PRMs:
  - **Quickly** generate a **small** roadmap of the Free Space F that has good **coverage** and **connectivity**

Probabilistic Road Maps

- PRMS have proven to useful in mapping free spaces that are difficult to model, or have many degrees of freedom.
  - This can facilitate fast planning for these situations
- Trade-off
  - PRMs often sacrifice **completeness** for **speed**

Probabilistic Road Maps

Moving Objects, Kindel

Probabilistic Road Maps

- Two Main Strategies:

1. Multi-Query:
   - Generate a single roadmap of *F* which can be used many times.
2. Single-Query:
   - Use a new roadmap to characterize the subspace of F which is relevant to the search problem.

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Motion Planning: Probabilistic Road Maps

- Single-Query PRMs (a.k.a. Rapidly Exploring Random Trees - RRTs)
  - Try to only sample a subspace of F that is relevant to the problem.
  - Probabilistically complete assuming C is *expansive* [Hsu et. al. 2000].
  - Very fast for many applications (allow for on-the-fly planning).

Motion Planning: Probabilistic Road Maps

- Two approaches:

1. Single Directional:
   - Grow a milestone tree from start configuration until the tree reaches the goal configuration

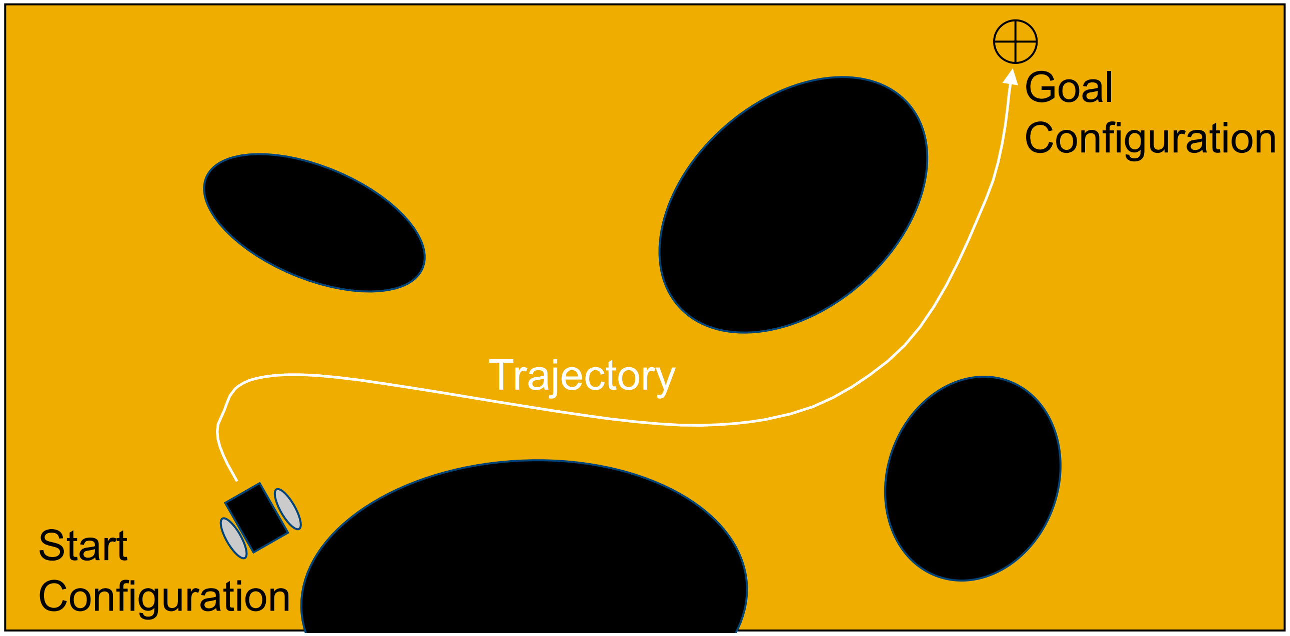
2. Bi-Directional:
   - Grow two trees, one from the start configuration and one from the goal configuration, until the two trees meet.
   - Can’t consider time in the configuration space

Single Query PRMs: Outline

1. Introduction
2. Algorithm Overview
3. Sampling strategies

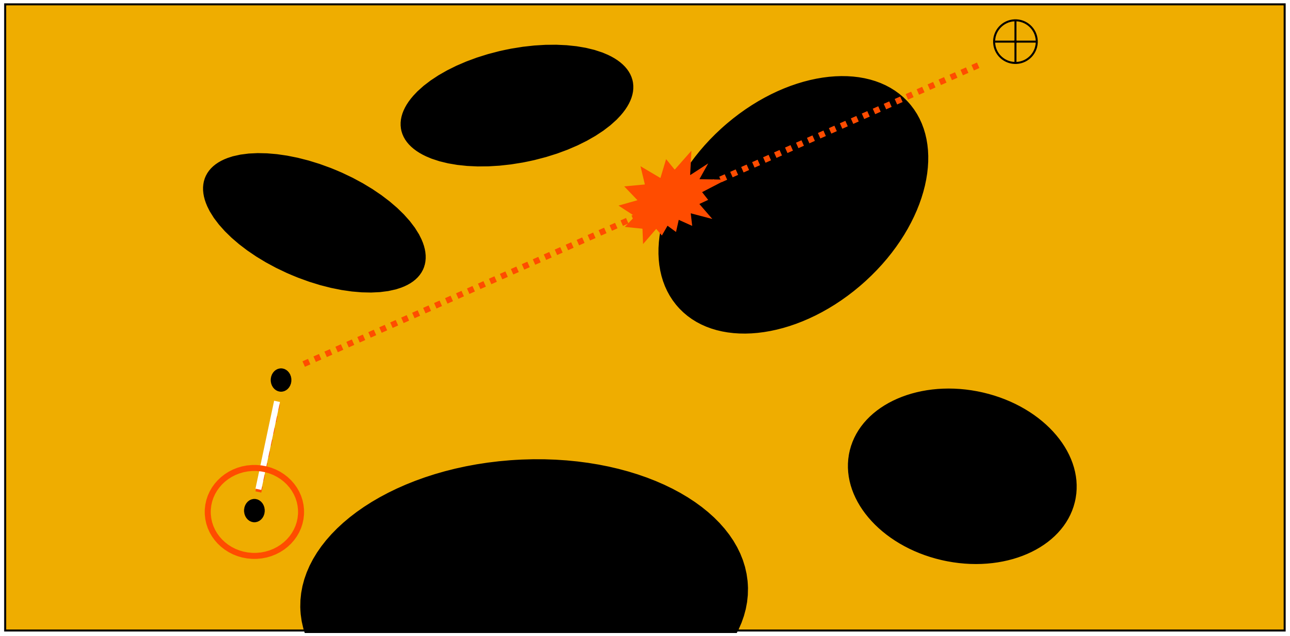
MP Overview

- Example:



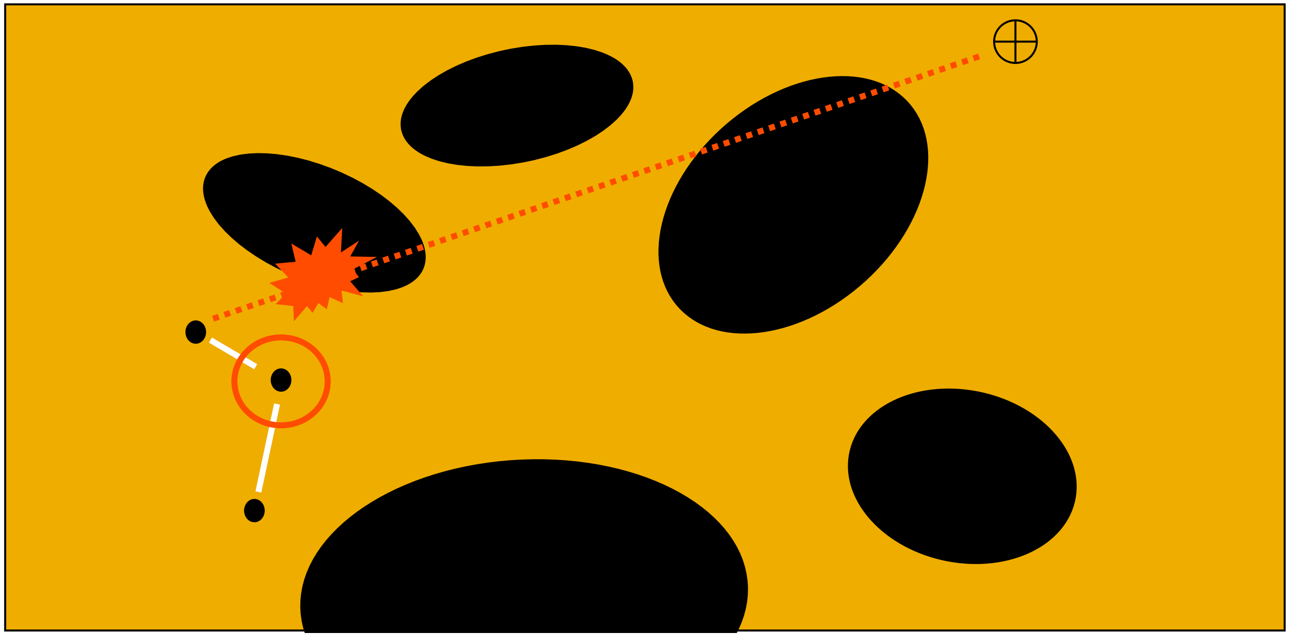
Motion Planning

- Example: Iteration 1



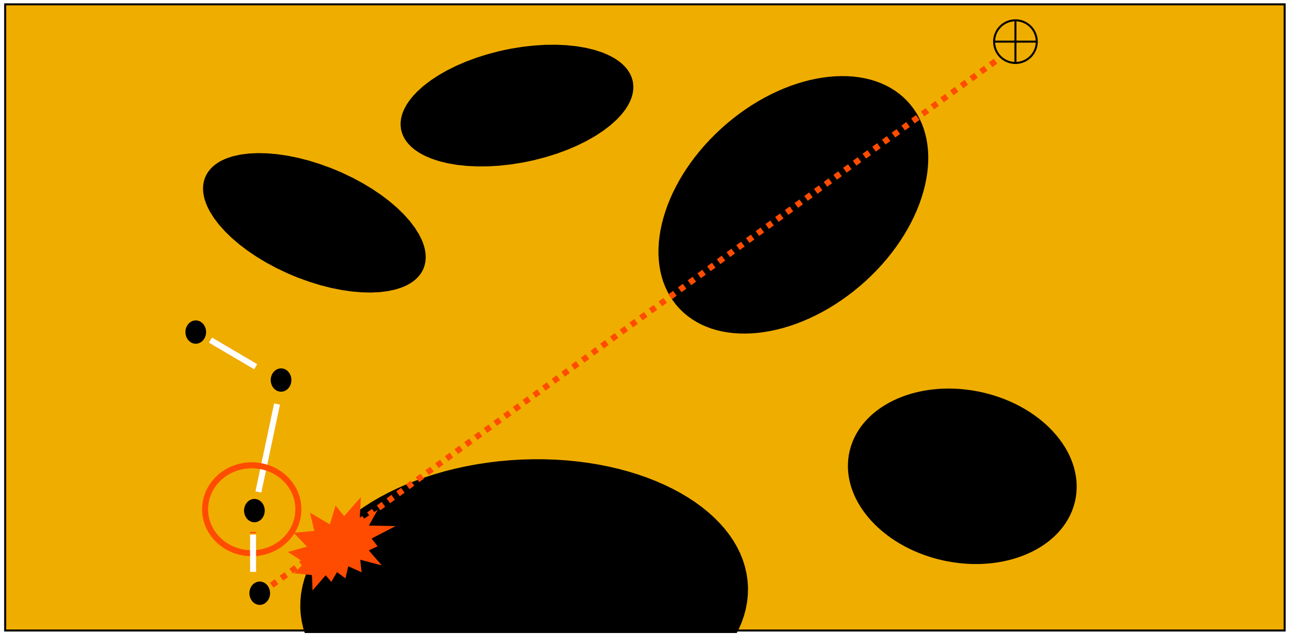
Motion Planning

- Example: Iteration 2



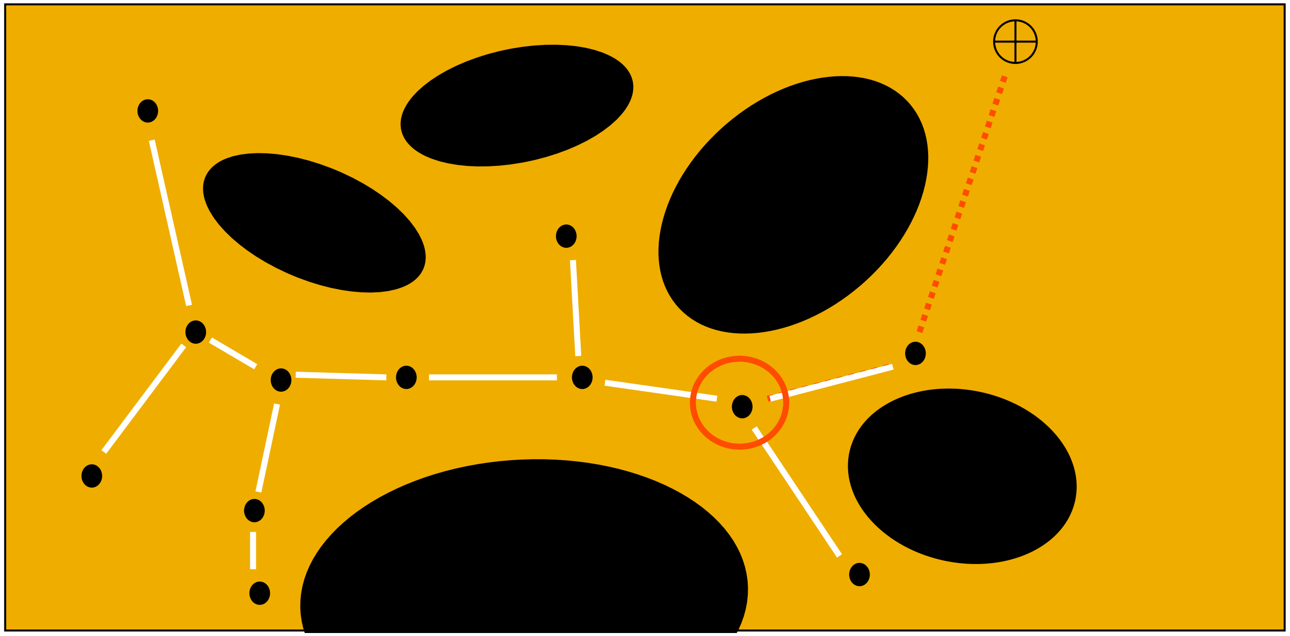
Motion Planning

- Example: Iteration 3



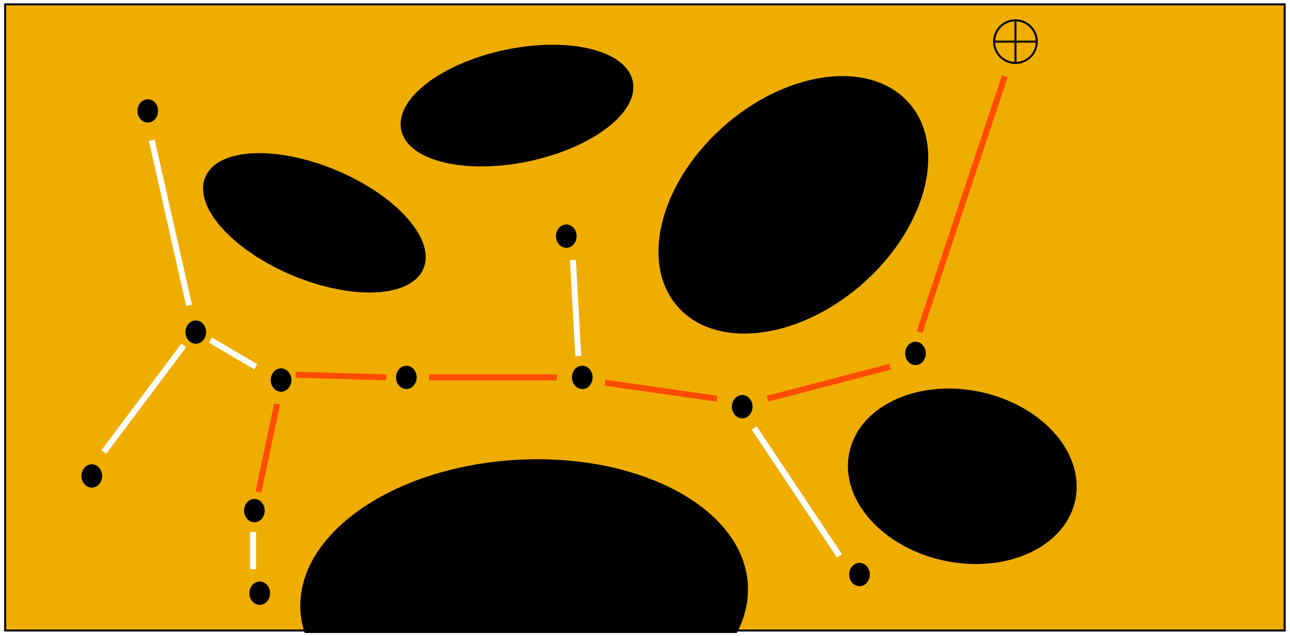
Motion Planning

- Example: Iteration 11



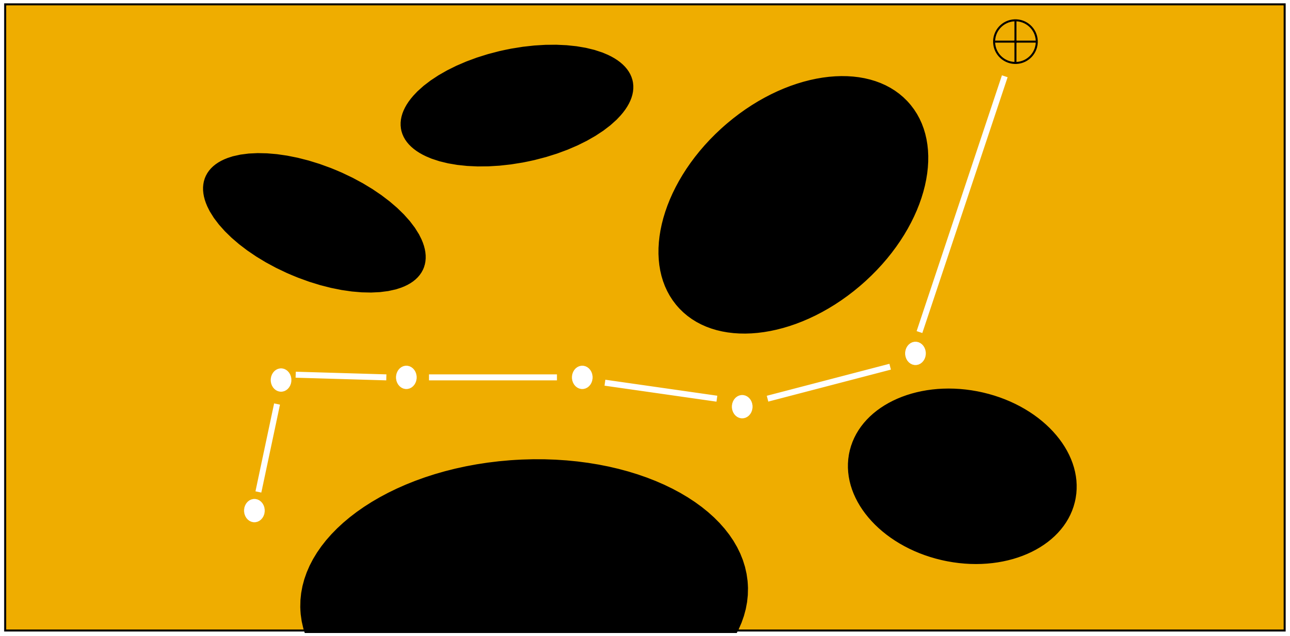
Motion Planning

- Example: Construct Path



Motion Planning

- Example: Construct Path



Probabilistic Road Maps: Learning Phase

- Nomenclature

$R = (\mathbf{N}, \mathbf{E})$	RoadMap
$\mathbf{N}$	Set of Nodes
$\mathbf{E}$	Set of edges
c	Configuration
e	edge

Motion Planning: Probabilistic Road Maps

- Algorithm
  1. Add start configuration c_{start} to $R(\mathbf{N}, \mathbf{E})$
  2. Loop
  3. Randomly Select New Node *c* to expand
  4. Randomly Generate new Node c' from c
  5. If edge e from c to c' is collision-free
  6. Add (c', e) to R
  7. If c' belongs to endgame region, return path
  8. Return if stopping criteria is met

Single Query PRMs: Outline

1. Introduction
2. Algorithm Overview
3. Sampling strategies
   - Node Selection (step 3)
   - Node Generation (step 4)
   - Endgame Region (step 7)

Motion Planning: PRM Node Selection

- One could pick the next node for expansion by picking from all nodes in the roadmap with equal probability.

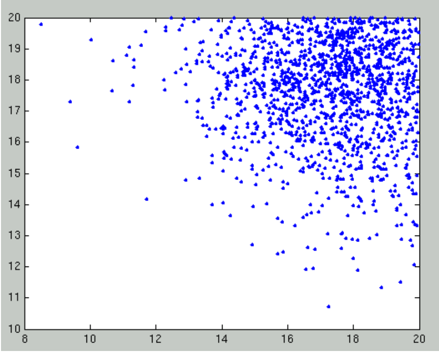
  - This is easy to implement, but leads to poor expansion ⟶ **Clustering**

Motion Planning: PRM Node Selection

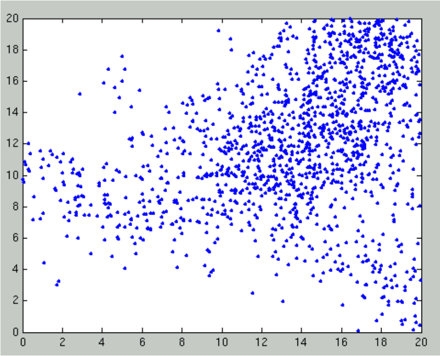
- Cont'
  - Method is to weight the random selection of nodes to expand, this can greatly affect the roadmap coverage of the configuration space.
  - Want to pick nodes with probability proportional to the inverse of node density.

Motion Planning: PRM Node Selection

- Example:
  - Presented is a 2DOF configuration space where the initial node in the roadmap is located in the upper right corner.
  - After X iterations, the roadmap produced from an unweighted expansion has limited coverage.



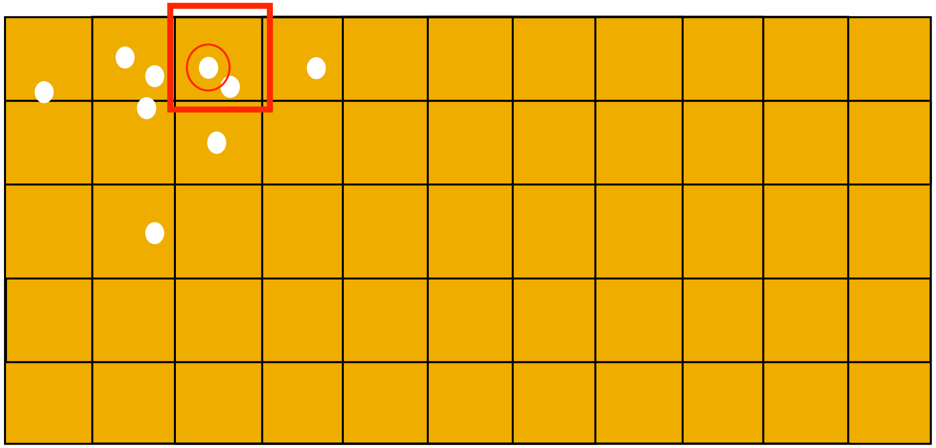
Unweighted



Weighted

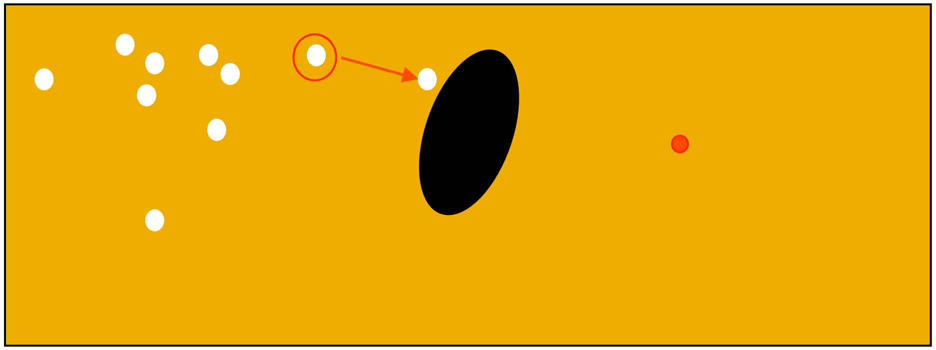
Motion Planning: PRM Node Selection Technique 1

- The workspace was divided up into cells to form a grid [Kindel 2000].
  - Algorithm:
    1. Randomly pick an occupied cell from the grid.
    2. Randomly pick a milestone in that cell.



Motion Planning: PRM Node Selection Technique 2

- Commonly used in Rapidly exploring Random Trees (RRTs) [Lavalle]
  - Algorithm:
    1. Randomly pick configuration c_{rand} from C.
    2. Find node c from R that is closest to node c_{rand}
    3. Expand from c in the direction of c_{rand}



Single Query PRMs: Outline

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   - Endgame Region (step 7)

Motion Planning: PRM Milestone Generation

- Use random control inputs to propagate robot from previous node c to new configuration c'

  - Algorithm:
    1. Randomly select controls u and Δt
    2. Use known dynamics/kinematics equation *f* of robot to generate new configuration

       $$c' = f(c, u, \Delta t)$$

    3. If path from c to c' is collision-free, then add c' to R

Motion Planning:
PRM Milestone Generation

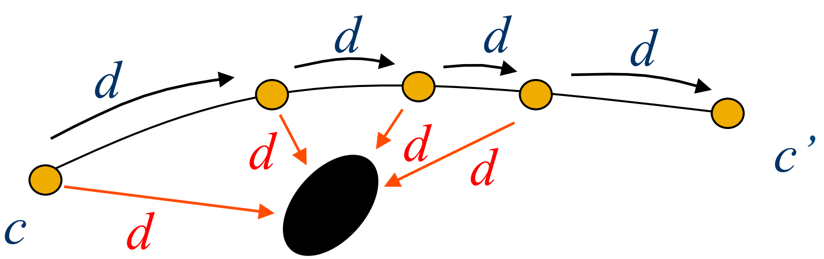
- Example: Differential drive robot
  1. Randomly select controls $\dot{\varphi}_{left}$, $\dot{\varphi}_{right}$ and Δt
  2. Propagate:
     1. Get Δs_{left} and Δs_{right}
     2. Calculate new state c' with:

$$c' = f(x, y, \theta, \Delta s_r, \Delta s_l) = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin\left(\theta + \frac{\Delta s_r - \Delta s_l}{2b}\right) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

     3. Use iterative search to check for collisions on path.

Motion Planning:
PRM Milestone Generation

- Example: Differential drive robot (cont’)
  - Iterative Collision checking is simple but not always efficient:
  - Algorithm:
    1. Calculate distance d to nearest obstacle
    2. Propagate forward distance *d* along path from c to c'
    3. If d is too small, return **collision**
    4. If c reaches or surpasses c', return **collision-free**



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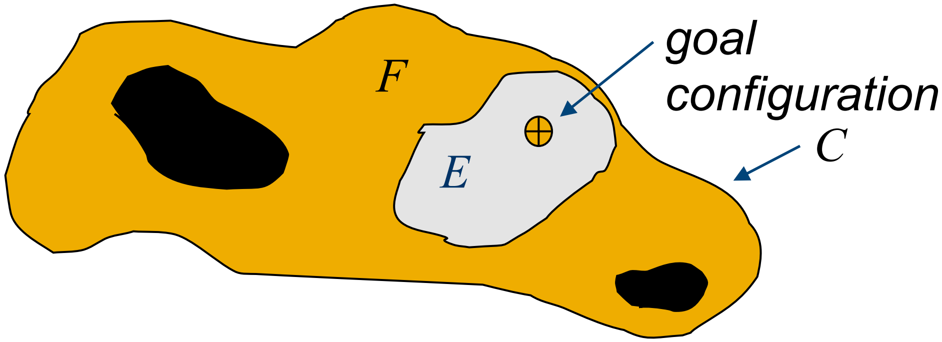
Motion Planning: PRM Endgame Region

- We define the endgame region E, to be the set of configurations that have a **simple** connection to the goal configuration.
  - For each planning problem, we can define a unique method of making **simple** connections.
  - This method will inherently define E.



Motion Planning: PRM Endgame Region

- Given the complexity of most configuration spaces, it is very difficult to model E.
  - In practice, we develop a simple admissibility test to calculate if a configuration c' belongs to the E
  - At every iteration of the algorithm, this test is used to determine if newly generated configurations are connected to the goal configuration.

Motion Planning: PRM Endgame Region

- In defining E, we need two things for good performance:

1. The region E should be **large**: this increases the chance that a newly generated milestone will belong to E and provide us a solution.
2. The admissibility test to be as **fast** as possible. This test is conducted at every iteration of the algorithm and will greatly affect the algorithm running time.

Motion Planning: PRM Endgame Region

- Several endgame definitions exist:
  1. The set of all configurations within some radius r of the goal configuration

Motion Planning: PRM Endgame Region

- Several endgame definitions exist:
  1. The set of all configurations within some radius r of the goal configuration
  2. The set of all configurations that have “simple”, collision-free connection with the goal configuration.
     - Example: Use circular arc for differential drive robots.