Chaotic Elastica and Taylor-Couette Flow

Physical Demos of Mathematical Concepts

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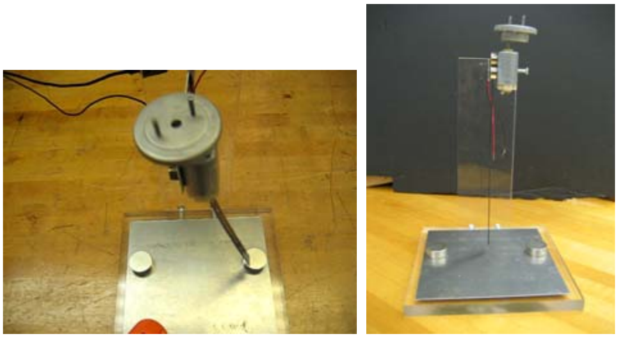
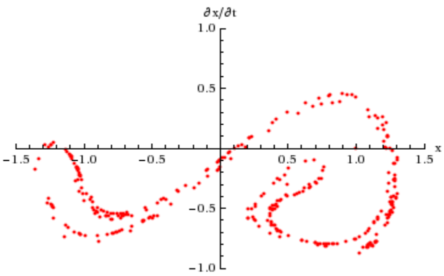


Chaotic pendulum obeying the driven Duffing equation.

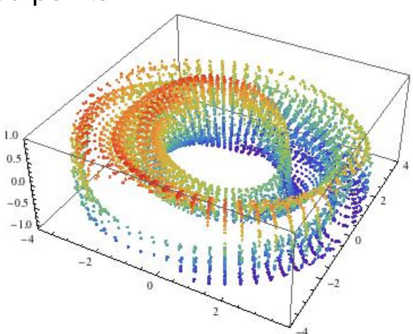


Poincaré section of Duffing equation with sampling period of π.

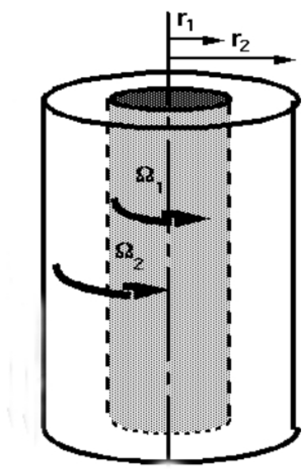
Goal: The goal of this project is to develop physical demos that can be modeled from the fields of nonlinear dynamics and partial differential equations. The elastica demos exhibit chaotic behavior when coupled with a periodic forcing source, while the Taylor-Couette demo shows fluid flow between two concentric cylinders. The demos illustrate concepts from chaos theory to PDEs, and will contribute to the growing collection of the HMC Math Demo Lab.

Outcome: Making the demos is easier said than done. Finding the right materials for building the demos and ensuring that the observed behaviors are consistent with what theory dictates is a challenge. For example, a 0.002'' difference in thickness for the chaotic beam results in a noticeable change in behavior.

Computational Investigation: The nonlinear equations governing the systems were also studied numerically using Mathematica. For example, order within the apparent chaos of oscillations in the elastica demos can be seen in Poincaré sections and fractal basins of attraction of fixed points.



Toroidal Poincaré map of Duffing Equation.



Future work: A more extensive analysis of the demos could provide us with better insight into their behaviors. Our project yielded results both in the physical and theoretical realms, but a marriage between the two would be a natural venue to explore in the future.

Sources:

Moon, Francis C. *Chaotic and Fractal Dynamics*. Wiley-Interscience, 1992.

Swinney, Harry, et al., Transition to shear-driven turbulence in Couette-Taylor flow. Phys Rev A (46), p. 6390 (1992).

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